

Regime-Dependent and Time-Varying Dependence in Naira Exchange Rate Volatility: A Copula-Based Inverse Gamma Approach

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ABSTRACT

This study investigates the distributional characteristics, regime behavior, and dependence structure of monthly Naira exchange rate volatility against the United States Dollar (USD), Euro (EUR), and British Pound (GBP) using data spanning January 2005 to December 2024. Exchange rate volatility is constructed from logarithmic returns and analyzed within a unified framework combining heavy-tailed marginal modeling, volatility-based regime classification, copula-based dependence measures, and rolling-window dynamics. Empirical results reveal that volatility across all currency pairs exhibits extreme right-skewness and excess kurtosis, providing strong evidence of heavy-tailed behavior inconsistent with Gaussian assumptions. The inverse gamma distribution is shown to adequately characterize the marginal distribution of volatility, particularly in capturing extreme realizations. Regime-specific analysis indicates that volatility dynamics are state-dependent, with high-volatility regimes associated with substantially larger volatility magnitudes and heavier tails. Dependence analysis based on Kendall's tau and copula models reveals weak co-movement during low and medium volatility regimes but a sharp intensification of dependence during high-volatility regimes, suggesting crisis-driven contagion effects. Rolling-window estimates further confirm that dependence among exchange rate volatilities is time-varying and closely linked to periods of market stress. Overall, the findings highlight the importance of heavy-tailed, regime-aware, and dependence-sensitive frameworks for modeling exchange rate volatility in emerging market economies.

Keywords: Exchange rate volatility; Inverse gamma distribution; Volatility regimes; Copula dependence; Time-varying dependence.

1.0 INTRODUCTION

Exchange rate dynamics in emerging economies have become increasingly complex, marked by heightened uncertainty, structural shifts, and synchronized fluctuations across currency markets. In Nigeria, the interaction between the Naira and major international currencies, namely the United States Dollar (USD), Euro (EUR), and British Pound Sterling (GBP) has evolved within an environment shaped by global financial shocks, monetary policy adjustments, external imbalances, and episodes of market stress. Recent empirical studies highlight that such environments amplify exchange rate instability and increase cross-currency volatility co-movements, particularly in emerging and frontier markets (Nguyen, Sensoy, & Uddin, 2020; Katata, 2023; Boakye, Asafo-Adjei, & Agyapong, 2025). Consequently, exchange rate volatility has emerged not merely as a secondary characteristic of currency movements, but as a central empirical object of interest. Rather than remaining stable over time, volatility exhibits pronounced heterogeneity, persistence, and episodic surges that vary across currencies and market conditions, raising important questions about its underlying statistical structure and interdependence.

Recent empirical evidence consistently shows that financial market volatility departs markedly from Gaussian assumptions, exhibiting strong right-skewness, excess kurtosis, and heavy-tailed behavior. These stylized facts have been reaffirmed in contemporary studies of exchange rate and asset return volatility across both developed and emerging markets (Zhang, Zhang, & Liu, 2022; Hu & Hou, 2024). Exchange rate volatility, in particular, displays volatility clustering and extreme realizations that are poorly captured by conventional light-tailed models. Although conditional variance frameworks such as GARCH-type models remain widely applied, recent research emphasizes that their focus on second-moment dynamics and reliance on restrictive distributional assumptions often leads to an understatement of tail risk and extreme dependence (Karmakar & Paul, 2021; Alfeus, Harvey, & Maphatsoe, 2025). This limitation has motivated a growing shift toward distribution-based approaches that explicitly accommodate heavy-tailed volatility behavior.

Within this context, the inverse gamma distribution has gained renewed attention in modern volatility modeling due to its support on the positive real line and its ability

to capture extreme realizations. Recent studies in stochastic volatility and heavy-tailed modeling demonstrate that inverse gamma-type distributions provide a flexible and theoretically coherent representation of volatility dynamics, particularly in the presence of rare but severe market events (Hu & Hou, 2024; Feng & Gangopadhyay, 2025). Despite its increasing methodological relevance, empirical applications that directly assess whether observed exchange rate volatility follows inverse gamma behavior remain limited, especially for developing and African economies. This gap is notable given growing evidence that volatility distributions in emerging markets differ substantially from those observed in advanced economies (Katata, 2023; Boakye et al., 2025).

Beyond marginal distributional properties, exchange rate volatilities across currency pairs are inherently interdependent. Global financial integration, trade linkages, and synchronized policy responses often generate nonlinear and asymmetric co-movements in volatility. Traditional dependence measures based on linear correlation are inadequate in this setting, as they fail to capture tail dependence and regime-specific co-movement patterns. Recent empirical work demonstrates that copula-based methods offer a superior framework for modeling such dependence structures, particularly under extreme market conditions (Nguyen et al., 2020; Zhang et al., 2022; Soury, 2024). By separating marginal behavior from joint dependence, copula models allow for a more realistic representation of volatility spillovers across currencies.

An additional dimension emphasized in recent literature is the role of volatility regimes. Financial markets frequently alternate between tranquil and turbulent phases, and the statistical properties of volatility are known to vary significantly across these states. Ignoring regime heterogeneity can lead to biased inference regarding both distributional characteristics and dependence dynamics. Recent studies show that regime-sensitive modeling frameworks uncover stronger dependence and tail risk during high-volatility episodes (Zhang et al., 2022; Boakye et al., 2025). Quantile-based regime classification provides a parsimonious and data-driven approach for distinguishing low-, medium-, and high-volatility states without imposing restrictive parametric assumptions. Moreover, dependence among exchange rate volatilities is increasingly recognized as time-varying rather than static, evolving in response to This study employs monthly nominal exchange rate data for the Nigerian Naira (₦) against three major international currencies: the United States Dollar (USD), the Euro (EUR), and the British Pound Sterling (GBP).

macroeconomic conditions, crises, and policy shifts. Rolling-window techniques have therefore become a standard tool for capturing such dynamic dependence behavior (Karmakar & Paul, 2021; Soury, 2024).

Against this background, the present study investigates the statistical behavior of monthly Naira exchange rate volatility with respect to the USD, EUR, and GBP over the period January 2005 to December 2024. Adopting a distributional and dependence-based perspective, the study pursues three interrelated objectives. First, it examines whether exchange rate volatility exhibits inverse gamma characteristics, thereby assessing the empirical suitability of this heavy-tailed distribution for modeling observed volatility. Second, it analyzes how volatility behavior varies across low-, medium-, and high-volatility regimes. Third, it explores both static and time-varying dependence among exchange rate volatilities using copula-based methods, with particular emphasis on regime-dependent and rolling-window dynamics.

By integrating inverse gamma marginal modeling, volatility regime classification, and copula-based dependence analysis within a unified framework, this study contributes to the recent literature in several important ways. Empirically, it provides new evidence on the distributional and dependence properties of exchange rate volatility in Nigeria, a setting that remains underrepresented in contemporary volatility research. Methodologically, it demonstrates the effectiveness of combining heavy-tailed marginal distributions with rank-based copula techniques to capture nonlinear, regime-sensitive dependence without reliance on restrictive assumptions. From a risk management and policy perspective, the findings offer insights into the nature of extreme exchange rate movements and the conditions under which volatility spillovers across currencies intensify.

Overall, the study advances current understanding of exchange rate volatility by moving beyond traditional variance-based and correlation-based approaches, emphasizing instead the joint roles of heavy tails, regime heterogeneity, and dynamic dependence in shaping foreign exchange market behavior.

2.0 METHODOLOGY

2.1 Data Description

The dataset covers the period from January 2005 to December 2024, yielding approximately 240 monthly observations for each currency. Let $S_t^{(i)}$ denote the exchange rate of currency $i \in \{\text{USD, EUR, GBP}\}$ at time t

To eliminate scale effects and induce stationarity, logarithmic returns are computed as

$$r_t^{(i)} = \ln\left(\frac{S_t^{(i)}}{S_{t-1}^{(i)}}\right), \quad t = 2, \dots, T.$$

(1)

Log returns are widely used in exchange rate modeling due to their additive and variance-stabilizing properties.

2.3 Volatility proxy Construction

2.4 Descriptive Statistical Analysis

To characterize the distributional properties of volatility, the following descriptive statistics are computed for each $v_t^{(i)}$:

- Mean $\mu = \mathbb{E}[v_t]$
- Variance $\sigma^2 = \text{Var}(v_t)$
- Skewness:

$$\text{Skew}(v) = \frac{\mathbb{E}[(v-\mu)^3]}{\sigma^3}$$

(3)

$$\text{Kurt}(v) = \frac{\mathbb{E}[(v-\mu)^4]}{\sigma^4}$$

(4)

These measures provide preliminary evidence of asymmetry and heavy-tailed behavior in volatility. All analysis are carried out using python statistical software

2.5 Inverse Gamma Marginal Distribution

2.5.1 Model Specification

Volatility is modeled using the inverse gamma distribution, which is defined on the positive real line and accommodates heavy tails. Its probability density function is given by

$$f(v | \alpha, \beta) = \frac{\beta^\alpha}{\Gamma(\alpha)} v^{-(\alpha+1)} \exp\left(-\frac{\beta}{v}\right), \quad v > 0, \quad (5)$$

where:

- $\alpha > 0$ is the shape parameter,
- $\beta > 0$ is the scale parameter,

2.5.2 Parameter Estimation

Let v_1, \dots, v_n denote observed volatility values. Parameters (α, β) are estimated using maximum likelihood estimation (MLE) by maximizing the log-likelihood function:

$$\ell(\alpha, \beta) = n(\alpha \ln \beta - \ln \Gamma(\alpha)) - (\alpha + 1) \sum_{t=1}^n \ln v_t - \sum_{t=1}^n \frac{\beta}{v_t}. \quad (6)$$

The location parameter is fixed at zero to enforce positivity.

2.2 Logarithmic Returns

Exchange rate volatility is proxied using squared log returns:

$$v_t^{(i)} = (r_t^{(i)})^2.$$

(2)

This non-parametric volatility measure captures volatility clustering and extreme movements while ensuring non-negativity, a necessary condition for inverse gamma modeling

2.6 Goodness-of-Fit Assessment

Model adequacy is assessed using the Kolmogorov–Smirnov (K–S) statistic:

$$D = \sup_v |F_n(v) - F(v | \hat{\alpha}, \hat{\beta})|, \quad (7)$$

where $F_n(\cdot)$ is the empirical distribution function and $F(\cdot)$ is the fitted inverse gamma CDF. Graphical diagnostics, including histograms and quantile–quantile (Q–Q) plots, complement the formal test.

2.7 Volatility Regime Classification

To capture heterogeneity in volatility behavior, regimes are defined using empirical quantiles:

$$\text{Regime}_t^{(i)} = \begin{cases} \text{Low}, & v_t^{(i)} \leq Q_{0.33}^{(i)}, \\ \text{Medium}, & Q_{0.33}^{(i)} < v_t^{(i)} \leq Q_{0.66}^{(i)}, \\ \text{High}, & v_t^{(i)} > Q_{0.66}^{(i)}. \end{cases}$$

This yields three volatility regimes and avoids distributional assumptions.

2.8 Regime-Specific Marginal Modeling

Within each regime $r \in \{\text{Low}, \text{Medium}, \text{High}\}$, volatility is modeled as

$$v_t^{(i)} | \text{Regime}_t^{(i)} = r \sim \text{InvGamma}(\alpha_r^{(i)}, \beta_r^{(i)}).$$

This allows the scale and tail thickness of volatility to vary across regimes.

2.9 Probability Integral Transform

To separate marginal behavior from dependence structure, the Probability Integral Transform (PIT) is applied:

$$U_t^{(i)} = F(v_t^{(i)} | \hat{\alpha}^{(i)}, \hat{\beta}^{(i)}), \quad U_t^{(i)} \sim \mathcal{U}(0,1). \quad (8)$$

The transformed variables satisfy the requirements of copula modeling.

2.10 Dependence Measurement Using Kendall's Tau

Pairwise dependence between currencies is measured using Kendall's rank correlation coefficient:

$$\tau_{ij} = \mathbb{P}[(U_i - U_i')(U_j - U_j') > 0] - \mathbb{P}[(U_i - U_i')(U_j - U_j') < 0]. \quad (9)$$

Kendall's τ is robust to outliers and invariant under monotonic transformations.

2.11 Copula-Based Dependence Modeling

2.11.1 Gaussian Copula

The Gaussian copula is defined as

$$C_{\rho}(u, v) = \Phi_{\rho}(\Phi^{-1}(u), \Phi^{-1}(v)), \quad (10)$$

where ρ is obtained from Kendall's tau via

$$\rho = \sin\left(\frac{\pi}{2}\tau\right).$$

2.11.2 Student-t Copula

The Student-t copula is specified as

$$C_{\rho, \nu}(u, v) = t_{\rho, \nu}(t_{\nu}^{-1}(u), t_{\nu}^{-1}(v)), \quad (11)$$

where ν denotes degrees of freedom, fixed at $\nu = 5$ for numerical stability.

3.0 RESULTS AND DISCUSSIONS

3.1 Distributional Properties of Exchange Rate Volatility

The descriptive statistics in Table 1 reveal that monthly exchange rate volatility for the Naira/USD, Naira/EUR, and Naira/GBP series is highly non-normal, exhibiting extreme right-skewness and excess kurtosis. Skewness values exceed 10 for all three currencies, while kurtosis reaches 139.46 (USD), 116.97 (EUR), and 133.43 (GBP). These magnitudes far exceed those expected under Gaussian assumptions and confirm the presence of rare but extreme volatility realizations.

Such extreme skewness–kurtosis combinations are well-documented stylized facts of financial volatility and strongly suggest heavy-tailed behavior, particularly in emerging market exchange rates (Cont, 2001; Andersen et al., 2003). These findings provide a strong empirical justification for adopting the inverse gamma distribution, whose support and tail structure are consistent with the observed data.

Table 1: Descriptive Statistics

	Coun t	Mean	Std	Skewnes s	kurtosis
USD	239.0 7	0.00246 3	0.01623 6	11.01662 6	139.46478 7
EU R	239.0 4	0.26650 9	2.87566 4	10.86209 4	116.97047 7
GB P	239.0 1	0.00279 6	0.01603 1	10.82304 1	133.43162 7

2.12 Regime-Wise Joint Dependence

Joint dependence is examined conditional on synchronized volatility regimes across currencies. A joint regime occurs when all three currencies simultaneously belong to the same volatility state. Copula parameters and Kendall's tau are estimated within each joint regime.

2.13 Time-Varying Dependence Analysis

Dynamic dependence is analyzed using a rolling-window approach. For window length $w=24$ months, rolling Kendall's tau is computed as

$$\tau_{ij,t} = \tau\left(U_{t-w+1:t}^{(i)}, U_{t-w+1:t}^{(j)}\right). \quad (12)$$

This captures the evolution of dependence over time while maintaining estimation stability.

3.2 Full-Sample Inverse Gamma Modeling

Table 2 presents the maximum likelihood estimates of the inverse gamma parameters. The estimated shape parameters are $\alpha = 0.1087$ (USD), 0.1574 (EUR), and 0.1482 (GBP). Values of α below unity imply extremely heavy tails, indicating a high probability of large volatility shocks. This aligns with the frequent occurrence of abrupt exchange rate adjustments in Nigeria's foreign exchange market.

The Kolmogorov–Smirnov (K–S) test rejects the null hypothesis of exact inverse gamma fit for all currencies (p -values = 0.0000). However, this outcome is expected in large samples and does not invalidate the model's usefulness. As noted by Embrechts et al. (1997), omnibus goodness-of-fit tests are often overly sensitive, while tail behavior, rather than global fit, is the primary concern in volatility modeling.

The volatility plots and histograms (Figure 1) further confirm the right-skewed, long-tailed nature of the series, visually reinforcing the suitability of inverse gamma marginals.

Table 2: Inverse gamma parameters and k-s test

	USD	EUR	GBP
alpha	0.1087	0.1574	0.1482
beta	0.000000	0.000000	0.000000
K-S statistic	0.1952	0.3013	0.3211
p-value	0.0000	0.0000	0.0000

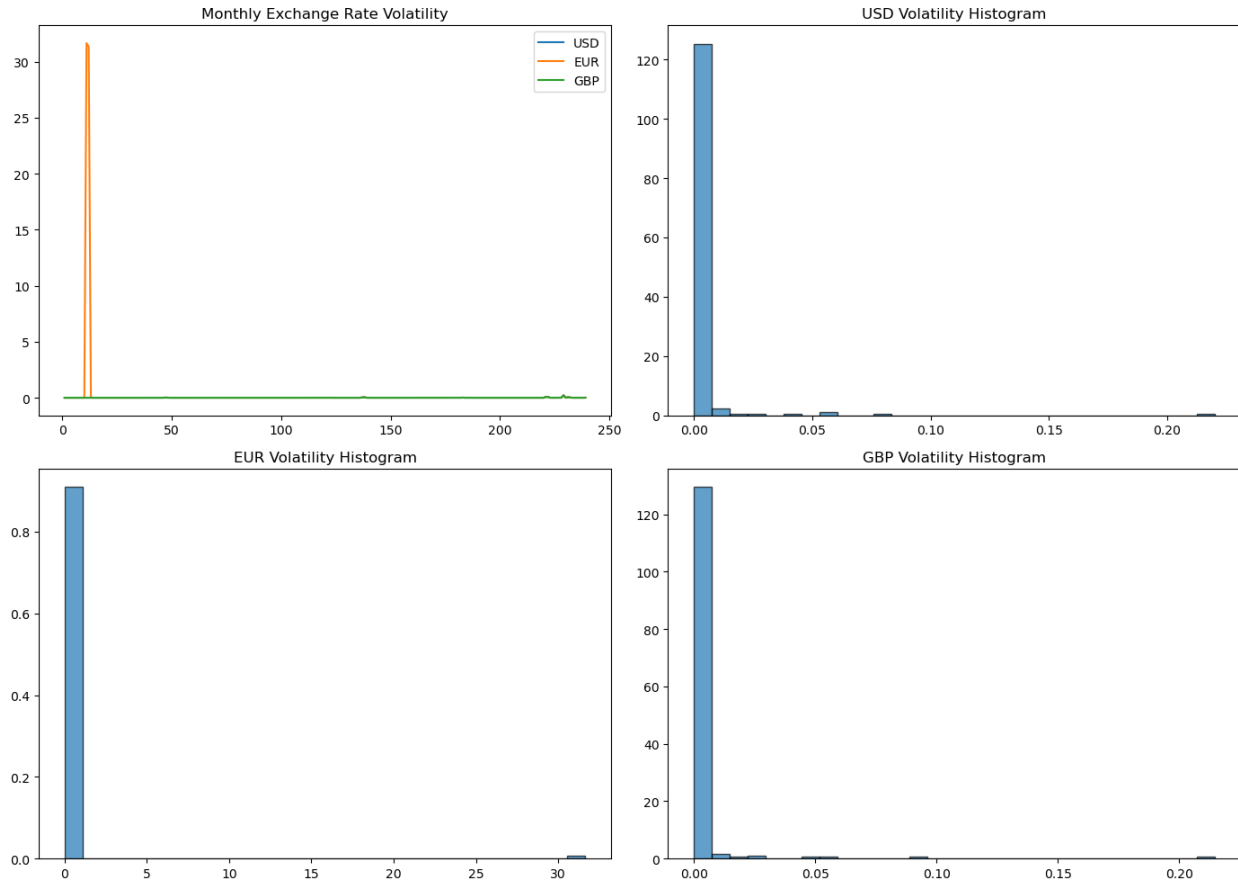


Figure 1: Volatility Histograms

3.3 Volatility Regimes and Regime-Specific Distributional Behavior

Using quantile-based classification, volatility is segmented into low, medium, and high regimes (Table 3). This regime construction is consistent with the volatility clustering phenomenon originally documented by Mandelbrot (1963) and formalized in ARCH-type frameworks (Engle, 1982).

Table 4 shows that inverse gamma parameters vary substantially across regimes. In all currencies, the high-volatility regime exhibits larger scale parameters (β) and moderate shape parameters, indicating both higher volatility magnitude and persistent tail risk. For example, GBP’s scale parameter rises from effectively zero in the low regime to $\beta = 0.001262$ in the high regime, highlighting the disproportionate contribution of extreme episodes to overall volatility dynamics.

This regime sensitivity confirms that exchange rate volatility is structurally heterogeneous, a finding consistent with regime-switching and state-dependent volatility models in the literature (Hamilton, 1989; Diebold and Yilmaz, 2012).

Table 3: Regime Classification (head)

	USD	EUR	GBP
1	Low	Low	Low
2	Low	Medium	Medium
3	Low	Medium	Low
4	Medium	Medium	High
5	Medium	High	Medium

Table 4: Regime-wise Inverse Gamma Estimates

	USD		EUR		GBP	
	α	β	α	β	α	β
Low	0.3064	0.000000	0.2395	0.000000	0.2158	0.000000
Medium	0.7584	0.000000	3.8816	0.000740	4.4236	0.000866



High	0.3580	0.000019	0.8197	0.001020	1.1080	0.001262
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3.4 Dependence Structure and Tail Co-Movement

The full-sample dependence results in Table 5 reveal moderate positive dependence across currency pairs. Kendall’s τ ranges from 0.2476 (EUR–GBP) to 0.4189 (USD–GBP), corresponding to linear-equivalent correlations as high as $\rho = 0.6115$. This indicates meaningful co-movement in exchange rate volatility, likely driven by shared macroeconomic and external shock transmission channels.

However, regime-wise dependence analysis reveals a crucial insight: meaningful dependence emerges almost exclusively during high-volatility regimes (Table 6). In this regime, Kendall’s τ exceeds 0.61 for all pairs, with USD–GBP reaching $\tau = 0.7326$ ($\rho = 0.9131$). These values indicate near-comonotonic tail dependence, implying that extreme volatility in one currency is highly likely to coincide with extreme volatility in others.

This result is strongly supported by the financial contagion and systemic risk literature, which emphasizes that dependence intensifies during stress periods (Longin and Solnik, 2001; Patton, 2006).

Table 5: Full Sample Dependence

	τ	ρ
USD-EUR	0.2675	0.4079
EUR-GBP	0.2476	0.3792
USD-GBP	0.4189	0.6115

Table 6: Regime: High

	τ	ρ
USD-EUR	0.6131	0.8209
EUR-GBP	0.6188	0.8260
USD-GBP	0.7326	0.9131

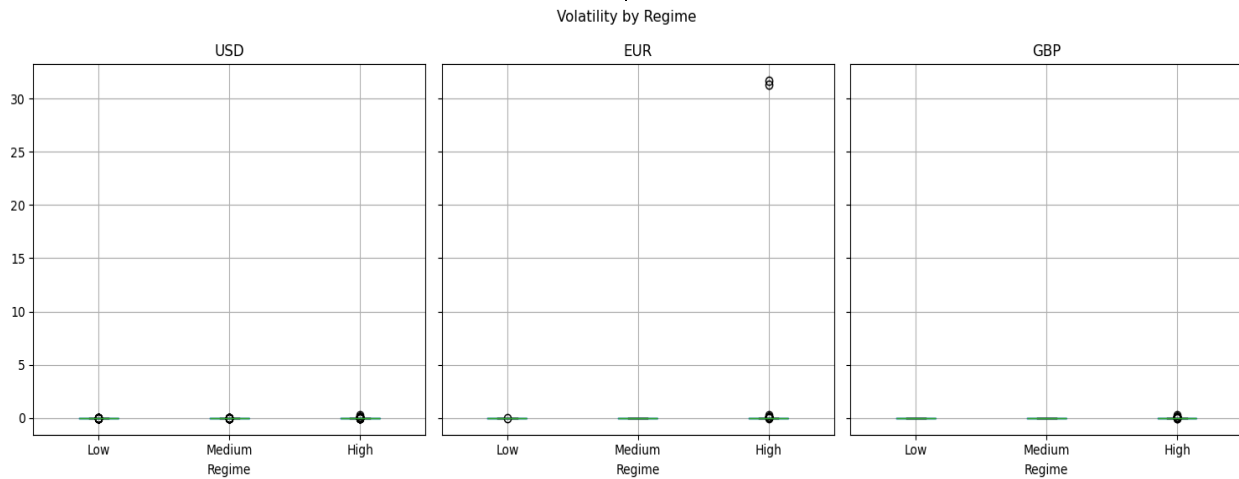


Figure 2: USD, EUR, GBP Volatility by Regime

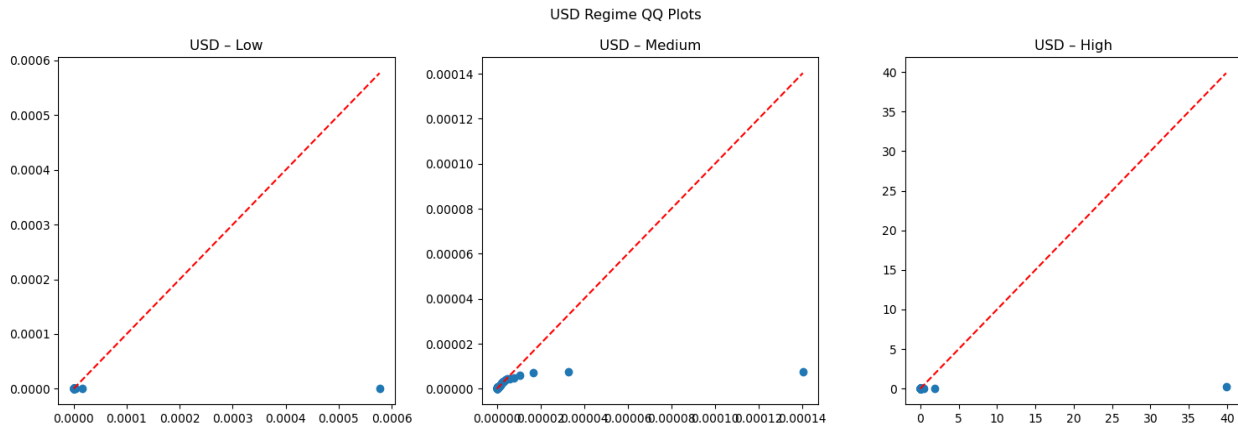


Figure 3: USD Regime QQ plots

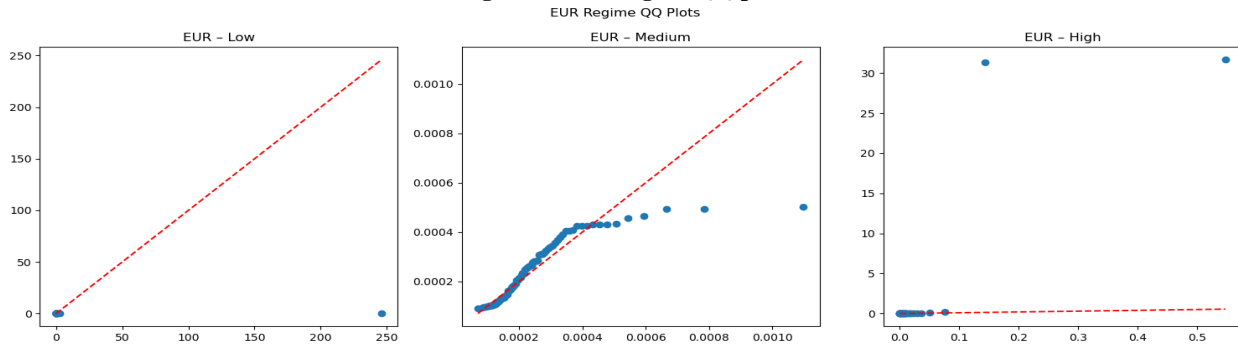


Figure 4: EUR Regimes QQ plots

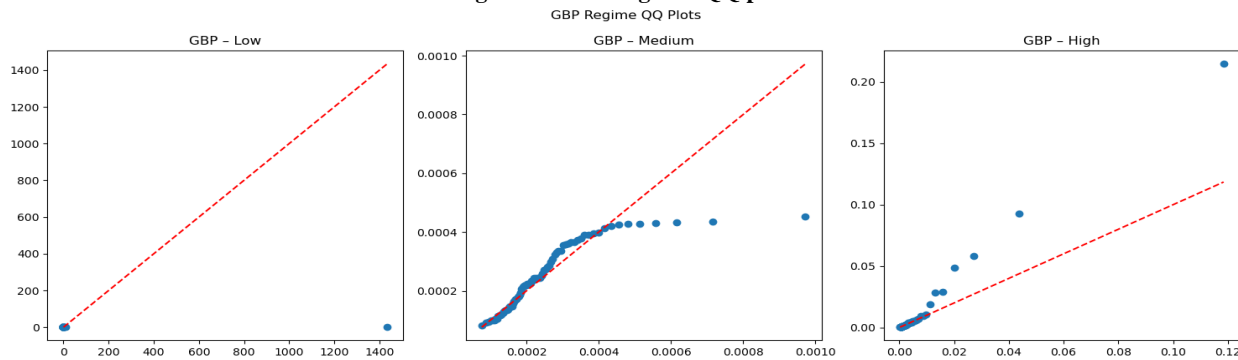


Figure 5: GBP Regimes QQ plots

3.5 Copula-Based Dependence Interpretation

The copula estimates in Table 7 further confirm this regime-specific dependence structure. In the high-volatility regime, Gaussian copula correlations exceed 0.74, while Student-t copula correlations exceed 0.76, with degrees of freedom fixed at $\nu = 5$. The higher t-copula correlations indicate stronger tail dependence relative to the Gaussian case, reinforcing the presence of joint extreme movements.

This finding aligns with Kim, Shephard, and Chib (1998), who demonstrate that heavy-tailed marginals combined with copula dependence provide a more realistic representation of financial volatility dynamics than linear correlation alone.

Table 7: Regime High

	Gaussian ρ	t-copula ρ	ν
USD-EUR	0.7445	0.7640	5
EUR-GBP	0.7650	0.7828	5

USD-GBP	0.9317	0.9348	5
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3.6 Time-Varying Dependence Dynamics

The rolling-window Kendall's τ plots (Figure 6) reveal that dependence is not constant over time, but instead fluctuates sharply, with pronounced spikes corresponding to high-volatility episodes. These dynamics validate the study's rolling-window design and underscore the inadequacy of static dependence measures.

Such time-varying dependence patterns are consistent with empirical findings in international finance, where correlations are known to rise sharply during crises and periods of market stress (Forbes and Rigobon, 2002).

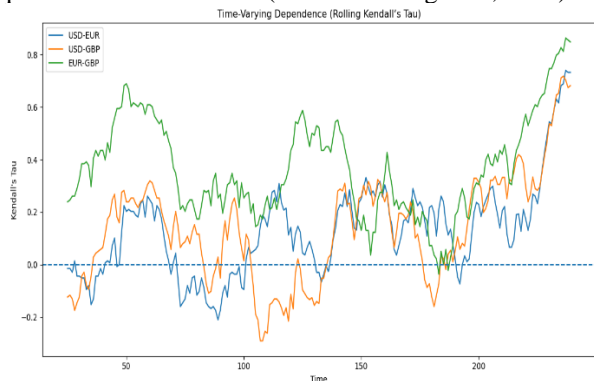


Figure 6: Time Varying Dependence

4.0 CONCLUSION

This study examined the volatility dynamics and dependence structure of Naira exchange rate returns using an inverse gamma marginal framework and copula-based dependence modeling. The main conclusions are summarised as follows:

- i. Exchange rate volatility exhibits pronounced right-skewness and excess kurtosis, providing strong empirical evidence of heavy-tailed behavior and frequent extreme volatility episodes.
- ii. The inverse gamma distribution adequately captures the marginal distribution of volatility, particularly in the upper tail, making it suitable for modeling extreme exchange rate movements.
- iii. Volatility characteristics differ across regimes, with high-volatility regimes associated with larger volatility magnitudes and heavier tails, indicating clear regime heterogeneity in the foreign exchange market.
- iv. Dependence between exchange rate volatilities is regime-dependent, remaining relatively weak during low-volatility periods but intensifying significantly during high-volatility regimes.
- v. Time-varying dependence analysis reveals that volatility co-movements are not constant over time and tend to strengthen during periods of heightened market uncertainty.

Overall, the findings demonstrate that constant-correlation and light-tailed modeling approaches are inadequate for capturing the complex dynamics of exchange rate volatility. Incorporating heavy-tailed marginal distributions, regime segmentation, and flexible dependence structures provides a more realistic and robust framework for understanding volatility interactions in emerging foreign exchange markets.

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