



**On Existence and Stability Analysis on Effects of Ivermectin Drugs:
Modeling the Dynamics and Control of Onchocerciasis Disease in Nigeria**

Jibril L¹, Amiru S², and Mansur M¹

¹Department of Mathematical Sciences, Federal University Gusau, Zamfara – Nigeria.

²Department of Mathematics, College of Education Maru, Zamfara – Nigeria.

Corresponding author: jibril.lawal@yahoo.com, +2348032202074

Received: November 2015

Revised and Accepted: November 2015

Abstract

The study incorporates the effects of ivermectin drugs, which effectively kills the onchocerca volvulus microfilariae in the human hosts. The study examined the existences and Equilibrium states of the model systems, and also carried out the stability analysis of the models equilibrium states using Bellman and Cooke's theorem. It was established that the state of total eradication of Onchocerciasis would be achieved and asymptotically stable in Nigeria if and only if, $J = F(0)G'(0) > 0$ whenever the ivermectin drugs and other control measures are maintained throughout the targeted period.

Keywords: *Onchocerciasis; Model; Microfilarie; Equilibrium; Stability.*

1.0 INTRODUCTION

Mexico is the world's third nation on July 29, 2015 to receive official verification of elimination of the disease from the World Health Organization (WHO, 2015). In September 2014, Ecuador becomes the second country in the world after Columbia in April 5, 2013 to be declared free of onchocerciasis after successfully implementing elimination for decades (WHO, 2014a). Onchocerciasis is a common, chronic, multisystematic disease caused by the nematode onchocerca volvulus. The disease characteristically includes dermatologic, lymphatic, ophthalmologic and systematic manifestations. Human transmission of the disease is caused by a

multiple bite from the intermediate host, the blackfly (genus simulium). Black flies breed along waterways, which can vary from small streams to broad rivers. Affected individuals usually live or work within a few kilometers where the vector breeds (Yaya *et al*, 2014). Until recently the blindness and skin pathology caused by heavy infections, has long been associated with a high incidence of detrimental effects on socioeconomic development and public health in endemic areas (WHO, 2014b). The disease is transmitted when a blackfly bites a person suffering from river blindness. When sucking the blood, tiny filarial worms are also taken in and the fly becomes

infected. On biting another person, it transfers these tiny worms which grow into adult worms. These adult worms go into lumps called nodules and there they reproduce other tiny worms. These tiny worms are then released to other parts of the body. Their movement under skin causes serious itching (Jibril and Ibrahim, 2011).

More than 99% of African onchocerciasis cases occur in 27 subs – Saharan countries, about 120 million people are at risk for infection in the continent (Medscape, 2015b). In West African savanna, the rate of infection reached as high as 80 - 100% between the ages of 20 years, with blindness peaking at age 40 – 50 years. In African forest areas, itching has affected 42% of the population older than 20 years and skin lesions have appeared in 28% of persons aged 5 years (Madscape, 2015a). About 50% of people older than 40 years may become blind in endemic areas, particularly in Africa, where females had a risk of death approximately 7.5% less than males. This is because men are affected more often than women as a result of involvements in farm and field occupation (Cooper *et al*, 1995). Nearly half of all river blindness diseases cases in the world are found in Nigeria. Approximately about 7 million Nigerians have the infection and 40 millions might have the disease if not urgently treated. Disease burden reported that onchocerciss is an important cause of blindness in many parts of northern Nigeria (Njepoume *et al*, 2009).

2.0 MATHEMATICAL FORMULATIONS

2.1 Model Assumptions

The epidemiological features of onchocerciasis lead to the following assumptions about the transmission dynamics of the disease:

- We assumed that both vector and human population are equally and are likely to be infected due to exposure to the vector breeding environment.
- That both infected and removed individual does not immigrate.
- That the new born are major source of human population recruitments.
- That the model ignores processes on parasite incubation periods in both populations.

The socio economic importance of the disease was the main reason for creation of the Onchocerciasis Control Programme in West Africa (OCP) in 1975, Onchocerciasis Elimination Programme in the America (OEPA) and the African Program for Onchocerciasis Control (APOC). Multiple programmes are working extensively to treat, to prevent and to eliminate onchocerciasis but onchocerciasis is still a very relevant world health problem (NTD, 2015). These programs rely on the regular distribution of ivermectin or mectizan drugs to lower the filarial load in affected individuals and thereby reduce the transmission and to mitigate the clinical manifestations of the infection. Ivermectin is an antimicrofilarial agent that acts as secondary prevention in infected individuals, reducing microfilarial survival and the burden of disease. It also provides primary prevention for the community by decreasing the number of microfilariae picked up by simulum during blood feedings, thus reducing transmission (CDC, 2014). This paper aims to formulate and to study the existence of the model system equations with effects of ivermectin drugs proposed by (Jibril and Ibrahim, 2011). And also to examine models equilibrium states together with the corresponding models characteristics equations and to analyzed the equilibrium states for the stability of onchocerciasis transmissions dynamics using Bellman and Cooke's criteria.

- That the model ignores the regulation of parasite population growth.
- That the individuals have annual free access to ivermectin drugs.
- That the natural death occurs in each population compartments.
- That the death occurs in infected human population class due to chronic river blindness.

2.2 Description of model parameters and variables

i) Host and Parasite Populations

H: Density of Human hosts.

V : density of Vectors (assumed to be constant).

$H_s(t)$: Compartment of human host susceptible at time t .

$H_I(t)$: Compartment of human host infective at time t .

$V_s(t)$: Compartment of vector population susceptible at time t .

$V_I(t)$: Compartment of vector population infective at time t .

ψ : Recruitment rate of human host susceptible (Births).

ii) Transmission Rates.

δ_H : Transmission rate from vector infective into human susceptible.

δ_V : Transmission rate from human infective into vector susceptible.

β : Movement rate out of human infective into human susceptible (recovery).

ξ : Movement rate out of human susceptible into human infective compartment.

α : Proportion of successfully ivermectin or mectizan cures the patient.

γ : Movement rate from vector susceptible into vector infective.

iii) Demographic Rates:

τ_H : Natural death of Human hosts susceptible.

τ_I : Natural death of Human hosts infective.

σ : Death of Human host infective as a result of chronic river blindness disease.

ρ_S : Death rate of vector susceptible.

ρ_I : Death rate of vector infective.

2.3 Mathematical Formulation of Model Equations

The following equations were derived based on the basic assumptions for the dynamics of the disease in the host population.

$$\left. \begin{aligned} \frac{dH_s(t)}{dt} &= \psi + \delta_H V_I + \alpha \beta H_I - \xi H_s V_I - \tau_H H_s \\ \frac{dH_I(t)}{dt} &= \xi H_s V_I - \alpha \beta H_I - \tau_I H_I - \sigma H_I - \delta_V H_I \\ \frac{dV_s(t)}{dt} &= \delta_V H_I - \rho_S V_s - \gamma V_s \\ \frac{dV_I(t)}{dt} &= \delta V_s - \rho_I V_I - \delta_H V_I \end{aligned} \right\} \tag{1}$$

2.4 Existence of Solutions to the Model System

As presented by (Akinyemi *et al*, 2015) to study the existence of HIV/AIDS models.

2.4.1 Statement of the theorem: Suppose we have the systems of equation below:

$$\left. \begin{aligned} y_1' &= f_1(t, y_1, y_2, \dots, y_n), & y_1(t_0) &= y_{10} \\ y_2' &= f_2(t, y_1, y_2, \dots, y_n), & y_2(t_0) &= y_{20} \\ &: \\ y_n' &= f_n(t, y_1, y_2, \dots, y_n), & y_n(t_0) &= y_{n0} \end{aligned} \right\} \quad (2)$$

Equation (2) can be written in a compact form as:

$$y_i' = f_i(t, y), \quad y(t_0) = y_0; \text{ where, } i = 1, 2, \dots, n \quad (3)$$

Theorem 1

Let Z denotes the region, such that

$$|t - t_0| \leq c, \text{ and } \|y - y_0\| \leq d, \text{ where, } y = (y_1, y_2, \dots, y_n) ; y_0 = (y_{10}, y_{20}, \dots, y_{n0}) \quad (4)$$

And suppose that $f(t, y)$ satisfies the Lipschitz condition

$$\|f(t, y_1) - f(t, y_2)\| \leq k \|y_1 - y_2\| \quad (5)$$

Whenever the pairs (t, y_1) and (t, y_2) belong to Z, where k is a positive constant $\delta > 0$ such that there exists a unique continuous vector solution $y(t)$ of the system (3) in the interval $|t - t_0| \leq \delta$. It is important to note that the

condition (5) is satisfied by the requirement that: $\frac{\partial f_i}{\partial x_j} ; i, j = 1, 2, \dots, n$

are continuous and bounded in Z. We investigate the existence and uniqueness of solution of the model systems around the region by satisfying:

1. The region of the interest is $0 \leq \omega \leq R$
2. A bounded solution of the form $0 \leq R \leq \infty$

are found in the region Z, whose partial derivatives satisfy $\delta \leq \omega \leq 0$, where ω and δ are positive constant.

Theorem 2

Let Z denote the region $0 \leq \omega \leq R$. Then the model systems (1) has a unique solution which is bounded and continuous in Z.

Proof: Let

$$\left. \begin{aligned} f_1 &= \psi + \delta_H V_I + \alpha \beta H_I - \xi H_S V_I - \tau_H H_S \\ f_2 &= \xi H_S V_I - \alpha \beta H_I - \tau_I H_I - \sigma H_I - \delta_V H_I \\ f_3 &= \delta_V H_I - \rho_S V_I - \gamma V_S \\ f_4 &= \delta V_S - \rho_I V_I - \delta_H V_I \end{aligned} \right\} \quad (8)$$

We show that $\frac{\partial f_i}{\partial x_j} ; i, j = 1, 2, \dots, n$ are continuous in Z. Then, the partial derivatives of systems (8) are

given below:

$$\left. \begin{aligned} \left| \frac{\partial f_1}{\partial H_S} \right| &= |-\xi V_I - \tau_H| < \infty; \left| \frac{\partial f_1}{\partial H_I} \right| = |\alpha\beta| < \infty; \left| \frac{\partial f_1}{\partial V_S} \right| = 0; \left| \frac{\partial f_1}{\partial V_I} \right| = |\delta_H - \xi H_S| < \infty \\ \left| \frac{\partial f_2}{\partial H_S} \right| &= |\xi V_I| < \infty; \left| \frac{\partial f_2}{\partial H_I} \right| = |-\alpha\beta - \tau_I - \delta_V - \sigma| < \infty; \left| \frac{\partial f_2}{\partial V_S} \right| = 0; \left| \frac{\partial f_2}{\partial V_I} \right| = |\xi H_S| < \infty \\ \left| \frac{\partial f_3}{\partial H_S} \right| &= 0; \left| \frac{\partial f_3}{\partial H_I} \right| = |\delta_V| < \infty; \left| \frac{\partial f_3}{\partial V_S} \right| = |-\gamma| < \infty; \left| \frac{\partial f_3}{\partial V_I} \right| = |-\rho_S| < \infty \\ \left| \frac{\partial f_4}{\partial H_S} \right| &= \left| \frac{\partial f_4}{\partial H_I} \right| = 0; \left| \frac{\partial f_4}{\partial V_S} \right| = |\delta| < \infty; \left| \frac{\partial f_4}{\partial V_I} \right| = |-\rho_I - \delta_H| < \infty \end{aligned} \right\} \quad (9)$$

It is distinct that all the partial derivatives in (9) are continuous and bounded. Hence, by theorem 1, there exist unique equations in (1).

3.1 Existence of Onchocerciasis Disease Free Equilibrium, E₀.

These are steady state solutions where there is no onchocerciasis disease presence in the human population.

At equilibrium point,

$$\frac{dH_S(t)}{dt} = \frac{dH_I(t)}{dt} = \frac{dV_S(t)}{dt} = \frac{dV_I(t)}{dt} = 0 \quad (10)$$

We let,

$$H_S(t) = x_1, \quad H_I(t) = x_2, \quad V_S(t) = x_3, \quad V_I(t) = x_4 \quad (11)$$

Substituting,

$$\left. \begin{aligned} \psi + \alpha\beta x_2 + \delta_H x_4 - (\xi x_4 + \tau_H) x_1 &= 0 \\ \xi x_1 x_4 - (\alpha\beta + \tau_I + \sigma + \delta_V) x_2 &= 0 \\ \delta_V x_2 - (\rho_S + \gamma) x_3 &= 0 \\ \gamma x_3 - (\rho_I + \delta_H) x_4 &= 0 \end{aligned} \right\} \quad (12)$$

Solving the model equations in (12) for the disease free equilibrium state. We obtained the Disease Free Equilibrium state as:

$$(x_1, x_2, x_3, x_4) = \left(\frac{\psi}{\tau_H}, 0, 0, 0 \right) \quad (13)$$

3.2 Existence of Onchocerciasis Endemic State Equilibrium, E_1 .

These are steady state solutions in which the onchocerciasis disease persists in the human population. We obtained the Endemic Equilibrium state as:

$$\left. \begin{aligned}
 x_1 &= \frac{(\rho_S + \gamma)(\rho_I + \delta_H)(\alpha\beta + \tau_I + \sigma + \delta_V)}{\xi\gamma\delta_V} \\
 x_2 &= \frac{(\rho_I + \delta_H)(\rho_S + \gamma) [\psi\xi\gamma\delta_V - \tau_H(\rho_S + \gamma)(\rho_I + \delta_H)(\alpha\beta + \tau_I + \sigma + \delta_V)]}{\gamma\delta_V [\xi(\rho_I + \delta_H)[(\rho_S + \gamma)(\tau_I + \sigma) + \rho_S\delta_V] + \xi\gamma\rho_I\delta_V} \\
 x_3 &= \frac{(\rho_I + \delta_H) [\psi\xi\gamma\delta_V - \tau_H(\rho_S + \gamma)(\rho_I + \delta_H)(\alpha\beta + \tau_I + \sigma + \delta_V)]}{\gamma [\xi(\rho_I + \delta_H)[(\rho_S + \gamma)(\tau_I + \sigma) + \rho_S\delta_V] + \xi\gamma\rho_I\delta_V} \\
 x_4 &= \frac{\psi\xi\gamma\delta_V - \tau_H(\rho_S + \gamma)(\rho_I + \delta_H)(\alpha\beta + \tau_I + \sigma + \delta_V)}{\xi(\rho_I + \delta_H)[(\tau_I + \sigma)(\rho_S + \gamma) + \rho_S\delta_V] + \xi\gamma\rho_I\delta_V}
 \end{aligned} \right\} \tag{14}$$

4.0 Stability Analysis of the Model Equations at Equilibrium Point.

On obtaining the equilibrium states, we then investigate the stability of the equilibrium states by examining the behavior of the model near the equilibrium states.

4.1 Stability Analysis of the Disease Free Equilibrium state.

We have already established that the systems of the model equations in (1) has the disease free equilibrium state,

$E_0 = \left(\frac{\psi}{\tau_H}, 0, 0, 0 \right)$. Then, the general variational matrix corresponding to the systems of model equations in (1) is:

$$J = \begin{pmatrix} -\tau_H & \alpha\beta & 0 & \delta_H - \xi\left(\frac{\psi}{\tau_H}\right) \\ 0 & -(\alpha\beta + \tau_I + \sigma + \delta_V) & 0 & \xi\left(\frac{\psi}{\tau_H}\right) \\ 0 & \delta_V & -(\rho_S + \gamma) & 0 \\ 0 & 0 & \gamma & -(\rho_I + \delta_H) \end{pmatrix} \tag{15}$$

Thus, this gives the Eigen – values:

$$\lambda_1 = -\tau_H, \lambda_2 = -\left(\frac{\psi\xi\gamma\delta_V}{\tau_H} + \alpha\beta + \tau_I + \sigma + \delta_V \right), \lambda_3 = -\left(\frac{\psi\xi\gamma\delta_V}{\tau_H} + \rho_S + \gamma \right), \lambda_4 = -\left(\frac{\psi\xi\gamma\delta_V}{\tau_H} + \rho_I + \delta_H \right) \tag{16}$$

Obviously, all the eigen – values $\lambda_1, \lambda_2, \lambda_3,$ and $\lambda_4,$ are negative, thus the systems of the model equations (1) is Locally Stable around the disease free equilibrium state, E_0 .

4.2 Stability Analysis of the Onchocerciasis Endemic Equilibrium state Using Bellman and Cooke’s Theorem.

4.2.1 Theorem 3: Bellman and Cooke’s Theorem

$M(Z) = P(Z, \ell^Z)$ Where $P(Z, Y)$; is a polynomial with principle terms; suppose $M(i, y), y \in \Re$ is separated into its real and imaginary parts $M(Z) = F(y) + iG(y)$, if all zeros of $M(z)$ have negative real parts, then the zeros of $F(y)$ and $G(y)$ are real, simple and alternate. Thus, $J = G'(0)F(0) - G(0)F'(0) > 0$, for all $y \in \Re$. Conversely, all zeros of $M(Z)$ will be in the left half

plane provided that either of the following condition are satisfied:

1. All the zeros of $F(y)$ and $G(y)$ are real, simple and alternate and inequality (17) is satisfy for at least one y .
2. All zeros of $F(y)$ are real, and for each zero, the relation (17) above is satisfied.
3. All zeros of $G(y)$ are real, and for each zero, the relation (17) above is satisfied.

Applying theorem (3) for stability analysis of the endemic equilibrium state as applied by (Momoh *et al*, 2012).

$$\begin{aligned}
 H(\lambda) = & \lambda^4 + \lambda^3(\tau_H + \delta_V + \rho_S + \gamma + \rho_I + \delta_H + \alpha\beta + \rho_S\rho_I + \tau_I + \sigma) + \lambda^2(\alpha\beta\tau_H + \rho_S\delta_H + \delta_V\tau_H + \gamma\rho_I \\
 & + \gamma\delta_H + \tau_I\tau_H + \sigma\tau_H + \tau_I\rho_S + \delta_V\rho_S + \tau_H\delta_H + \sigma\delta_H + \tau_H\rho_S + \delta_V\delta_H + \sigma\rho_S + \gamma\rho_I + \gamma\delta_H + \tau_I\delta_H + \tau_H\rho_I + \\
 & \alpha\beta\rho_S + \alpha\beta\delta_H + \tau_H\gamma + \alpha\beta\gamma + \alpha\beta\rho_I + \sigma\rho_I + \sigma\gamma + \delta_V\gamma + \delta_V\rho_I) + \lambda(\delta_V\tau_H\rho_S + \delta_V\tau_H\delta_H + \tau_H\rho_S\rho_I + \tau_H\rho_S\delta_H \\
 & + \tau_H\gamma\rho_I + \tau_H\gamma\delta_H + \alpha\beta\rho_S\rho_I + \alpha\beta\rho_S\delta_H + \alpha\beta\gamma\rho_I + \alpha\beta\gamma\delta_H + \alpha\beta\tau_H\rho_I + \tau_I\rho_S\rho_I + \tau_I\rho_S\delta_H + \tau_I\gamma\rho_I + \tau_I\gamma\delta_H + \\
 & \sigma\rho_S\rho_I + \sigma\rho_S\delta_H + \sigma\gamma\rho_I + \sigma\gamma\delta_H + \delta_V\rho_S\rho_I + \delta_V\rho_S\delta_H + \delta_V\gamma\rho_I + \alpha\beta\tau_H\delta_H + \delta_V\tau_H\rho_I + \alpha\beta\tau_H\rho_S + \alpha\beta\tau_H\gamma \\
 & + \tau_I\tau_H\rho_S + \tau_I\tau_H\delta_H + \tau_I\tau_H\rho_I + \tau_I\tau_H\gamma + \sigma\tau_H\rho_S + \sigma\tau_H\delta_H + \sigma\tau_H\gamma + \sigma\tau_H\rho_I + \delta_V\gamma\delta_H + \delta_V\tau_H\gamma) + \tau_I\tau_H\rho_S\delta_H \\
 & + \tau_I\tau_H\rho_S\rho_I + \tau_I\tau_H\gamma\rho_I + \tau_I\tau_H\gamma\delta_H + \sigma\tau_H\rho_S\rho_I + \sigma\tau_H\rho_S\delta_H + \sigma\tau_H\gamma\rho_I + \sigma\tau_H\gamma\delta_H + \delta_V\tau_H\rho_S\rho_I + \delta_V\tau_H\rho_S\delta_H \\
 & + \delta_V\tau_H\gamma\rho_I + \delta_V\tau_H\gamma\delta_H + \alpha\beta\tau_H\gamma\delta_H + \alpha\beta\tau_H\rho_S\rho_I + \alpha\beta\tau_H\rho_S\delta_H + \alpha\beta\tau_H\gamma\rho_I + \frac{\psi\xi\gamma\delta_V}{\tau_H} \tag{18}
 \end{aligned}$$

Setting $\lambda = iw$ (19)

$$\begin{aligned}
 H(\lambda) = H(iw) = & (iw)^4 + (iw)^3[(\tau_H + \delta_V + \rho_S + \gamma + \rho_I + \delta_H + \alpha\beta + \rho_S\rho_I + \tau_I + \sigma)] + (iw)^2[(\alpha\beta\tau_H + \rho_S\delta_H \\
 & + \delta_V\tau_H + \gamma\rho_I + \gamma\delta_H + \tau_I\tau_H + \sigma\tau_H + \tau_I\rho_S + \delta_V\rho_S + \tau_H\delta_H + \sigma\delta_H + \tau_H\rho_S + \delta_V\delta_H + \sigma\rho_S + \gamma\rho_I + \gamma\delta_H + \tau_I\delta_H \\
 & + \tau_H\rho_I + \alpha\beta\rho_S + \alpha\beta\delta_H + \tau_H\gamma + \alpha\beta\gamma + \alpha\beta\rho_I + \sigma\rho_I + \sigma\gamma + \delta_V\gamma + \delta_V\rho_I)] + (iw)[(\delta_V\tau_H\rho_S + \delta_V\tau_H\delta_H + \tau_H\rho_S\rho_I \\
 & + \tau_H\rho_S\delta_H + \tau_H\gamma\rho_I + \tau_H\gamma\delta_H + \alpha\beta\rho_S\rho_I + \alpha\beta\rho_S\delta_H + \alpha\beta\gamma\rho_I + \alpha\beta\gamma\delta_H + \alpha\beta\tau_H\rho_I + \tau_I\rho_S\rho_I + \tau_I\rho_S\delta_H + \tau_I\gamma\rho_I + \\
 & \tau_I\gamma\delta_H + \sigma\rho_S\rho_I + \sigma\rho_S\delta_H + \sigma\gamma\rho_I + \sigma\gamma\delta_H + \delta_V\rho_S\rho_I + \delta_V\rho_S\delta_H + \delta_V\gamma\rho_I + \alpha\beta\tau_H\delta_H + \delta_V\tau_H\rho_I + \alpha\beta\tau_H\rho_S + \\
 & \alpha\beta\tau_H\gamma + \tau_I\tau_H\rho_S + \tau_I\tau_H\delta_H + \tau_I\tau_H\rho_I + \tau_I\tau_H\gamma + \sigma\tau_H\rho_S + \sigma\tau_H\delta_H] \tag{20}
 \end{aligned}$$

Rearranging into real and imaginary parts, we have

$$H(iw) = F(w) + iG(w) \tag{21}$$

$$\begin{aligned}
 F(W) = & w^4 + w^2[(\alpha\beta\tau_H + \rho_S\delta_H + \delta_V\tau_H + \gamma\rho_I + \gamma\delta_H + \tau_I\tau_H + \sigma\tau_H + \tau_I\rho_S + \delta_V\rho_S + \tau_H\delta_H + \sigma\delta_H + \tau_H\rho_S + \\
 & \delta_V\delta_H + \sigma\rho_S + \gamma\rho_I + \gamma\delta_H + \tau_I\delta_H + \tau_H\rho_I + \alpha\beta\rho_S + \alpha\beta\delta_H + \tau_H\gamma + \alpha\beta\gamma + \alpha\beta\rho_I + \sigma\rho_I + \sigma\gamma + \delta_V\gamma + \delta_V\rho_I)] \\
 & + \tau_I\tau_H\rho_S\delta_H + \tau_I\tau_H\rho_S\rho_I + \tau_I\tau_H\gamma\rho_I + \tau_I\tau_H\gamma\delta_H + \sigma\tau_H\rho_S\rho_I + \sigma\tau_H\rho_S\delta_H + \sigma\tau_H\gamma\rho_I + \sigma\tau_H\gamma\delta_H + \delta_V\tau_H\rho_S\rho_I \\
 & + \delta_V\tau_H\rho_S\delta_H + \delta_V\tau_H\gamma\rho_I + \delta_V\tau_H\gamma\delta_H + \alpha\beta\tau_H\gamma\delta_H + \alpha\beta\tau_H\rho_S\rho_I + \alpha\beta\tau_H\rho_S\delta_H + \alpha\beta\tau_H\gamma\rho_I + \frac{\psi\xi\gamma\delta_V}{\tau_H} \tag{22}
 \end{aligned}$$

From the same condition given by the inequality:
 $G'(0)F(0) - G(0)F'(0) > 0$ (32)

Substituting equation (28) and (29) in (32)

i.e. $G(0) = 0$ and $F'(0) = 0$

Hence, equation (32) becomes;

$$F(0)G'(0) > 0 \tag{33}$$

We let, $J = F(0)G'(0)$ (34)

Hence, $J > 0$ (35)

Obviously, the condition in theorem (3) holds. Therefore, the onchocerciasis endemic equilibrium state for the model systems is Locally Asymptotically Stable (LAS).

5.0 NUMERICAL SIMULATION METHODS AND DISCUSSION OF RESULTS

Using hypothetical values for parameters of J we used mathematical software (maple) to evaluate J and the results are presented in the Table 1.

TABLE 1 Stability Analysis of the onchocerciasis model at endemic equilibrium state using Bellman and Cooke's theorem

ζ	α	β	γ	σ	δ_s	δ_i	ψ	τ_H	τ_I	δ_H	δ_V	$F(0)$	$G'(0)$	J	Remark
0.5	0.0	0.0		0.1	0.1	0.2	0.25	0.1	0.15	0.0	0.0	0.00350	0.0715	0.00025025	Stable
0.5	0.1	0.1	0.6	0.1	0.1	0.2	0.25	0.1	0.15	0.1	0.1	0.0826	0.1326	0.01095276	Stable
0.5	0.2	0.2	0.6	0.1	0.1	0.2	0.25	0.1	0.15	0.2	0.2	0.1637	0.2191	0.03586667	Stable
0.5	0.3	0.3	0.6	0.1	0.1	0.2	0.25	0.1	0.15	0.3	0.3	0.2474	0.3358	0.08307692	Stable
0.5	0.4	0.4	0.6	0.1	0.1	0.2	0.25	0.1	0.15	0.4	0.4	0.3340	0.4875	0.16282500	Stable

5.2 Discussion of Results

Clearly from the Table 1 above, we obtained the same remarks for $F(0)G'(0) > 0$ the value for J is greater than zero throughout, implying that the onchocerciasis disease is locally stable as far as $J > 0$. Therefore, the disease will dies out over the targeted period of time. From the Table 1 above we

obtained the most effective parameters that would lead to total eradication of the disease at $\zeta = 0.0$, $\alpha = 0.0$, $\beta = 0.0$, $\gamma = 0.6$, $\sigma = 0.1$, $\delta_s = 0.1$, $\delta_i = 0.1$, $\psi = 0.25$, $\tau_H = 0.1$, $\tau_I = 0.15$, $\delta_H = 0.0$, $\delta_V = 0.0$. On computation we obtained the value for J as 0.00025025. Hence, the conditions proposed in the theorem (3) hold.

5.3 Conclusions

The study presents four compartmental deterministic models. The existence and stability on effects of ivermectin drugs for control and spread of onchocerciasis disease in Nigeria were analyzed. The model systems were found to be continuous and bounded. The equilibrium states for the disease free

and endemic equilibrium states were examined. We analyzed the stability of the endemic equilibrium state using Bellman and cooke's theorem and its found to be Locally Asymptotically Stable (LAS), whenever $J > 0$ and otherwise, whenever $J < 0$. Therefore, the treatment of Onchocerciasis disease with ivermectin drugs is an effective treatment strategy that can potentially lead to eradication of the

disease by the year 2020 in Nigeria. Similarly, the following measures and control strategies would be helpful: identification of infected communities followed by treatment with ivermectin drugs; interruption of transmission by the destruction of

simulium (blackfly) through the use of insecticides e.g. aerial spraying to destroy blackfly larvae in rivers and streams; avoiding simuliun bites by covering parts of the body most at risk and avoiding areas where black flies breed.

6.0 REFERENCES

- Akinyemi S.T, Ibrahim M. O, Dogo M.M, Bakare N.G, (2015). *On Existence of a Staged Progression HIV/ AIDS Model with Control Measures*. Journal of Nigerian Association of Mathematical Physics. 29: pp 163 – 166.
- Center for Disease Control (CDC, 2014). *Parasites Onchocerciasis (River Blindness)*. Available at <http://www.cdc.gov/parasites/onchocerciasis/disease.html>.
- Cooper P.J, Proano R, Beltran C, Anselmi N, Guderian R.H, (1995). *Onchocerciasis in Ecuador: Ocular Findings in Onchocerca Volvulus Injected Individuals*. Br Journal of Ophthalmology. 79(2):157 – 62.
- Jibril L. and Ibrahim M.O, (2011). *Mathematical Modeling on the CDTI Prospects for Elimination of Onchocerciasis: A deterministic Model Approach: Research J of Mathematics and Statistics (RJMS-MSO)*. 3(4): pp 136 – 140.
- Medscape, (2015a). *Dermatologic Manifestations of Onchocerciasis (River Blindness)*. Available at <http://www.emedicine.medscape.com/article/1204593-overview>.
- Medscape, (2015b). *Ophthalmologic Manifestations of Onchocerciasis*. Available at <http://www.emedicine.medscape.com/article>.
- Momoh A.A, Ibrahim M.O, Madu B.A, Asogwa K.K, (2012). *Global Equilibrium Stability of Hepatitis B Model and Vaccination Impact*. Research Journal of Mathematics and statistics (RJMS-MSO). 4(3): 57 – 62.
- Njepoume N, Ogbu – Pearce P, Okoronkwo C, Igbe M.A, (2009). *Controlling Onchocerciasis: The Nigerian Experience*. The internet Journal of Parasite disease. 4: 1.
- Neglected Tropical Disease, (NTD, 2015). *Control and Elimination Program*. Available at <http://www.neglectedtropicaldisease.gov/target-disease/onchocerciasis>.
- World Health Organisation, (WHO, 2014a). *Prevention of Blindness and Visual Impairment: Onchocerciasis Control Programme (OCP)*. Available at <http://www.who.int/blindness/partnership/onchocerciasis-ocp/en/>.
- World Health Organisation, (WHO, 2014b). *A New Drop for River? Research training in Tropical disease (TDR) News*. WHO TDR News. Available at <http://www.who.int/tdr/publications/documents/tdrnews-issue-79.pdf>.
- World Health Organisation, (WHO, 2015). *Newest Victory in Onchocerciasis Elimination Mexico*. Available at <http://www.mectizan.org/news/newest.victory>. Accessed on 22nd November, 2015.
- Yaya G, Kobangué L, Kemata B, Galle D, Gresenguet G, (2014). *Elimination or Control of the Onchocerciasis in Africa? Case of Gami Village in Central African Republic*. Bulletin of the Society for Pathology and Exotique. 107(3): pp 188 – 93.