

**INTERNATIONAL JOURNAL OF SCIENCE FOR GLOBAL SUSTAINABILITY****Magnetohydrodynamic (MHD) Convective Flow in a Porous Medium in the Presence Variable Pressure and Injection/Suction**Salisu Saleh^{1,3}, Aishatu A. Halliru², Nasir M. Sarki³, Omokhuale Emmanuel⁴¹Department of Mathematics, Faculty of Science, Zamfara State University Talata Mafara, Zamfara State, Nigeria²Department of Mathematics, Faculty of Science, Bayaro University Kano, Nigeria³Department of Mathematics, Faculty of Science, Kebbi State University of Science and Technology Aliero, Kebbi, Nigeria⁴Department of Mathematics, Faculty of Science, Federal University Gusau, Zamfara State, NigeriaCorresponding Author's Email: salehsalisuag@gmail.com

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ABSTRACT

This article investigates MHD convective flow with variable pressure and injection/suction in a porous medium. The governing equations are modeled using partial differential equations (PDEs) in dimensional form. The PDEs describes the unsteady flow of an incompressible electrically conducting fluid are expressed for the momentum, energy and concentration of the fluid flow. Also, the PDEs were transformed to non-dimensional form using some dimensional variables. Perturbation method was used to obtain the solutions for the velocity, temperature and concentration. The non-dimensional Skin friction, Nusselt number and Sherwood number were obtained. Effects of physical parameters associated with the present study was examined and discussed. It was observed that the concentration, temperature and velocity of the fluid rises with increase in injection parameter and an opposite trend was observed when suction parameter became significant. It is found that the temperature of the fluid reduces as the Prandtl number increase. However, an opposite trend is seen for heat source/sink.

Keywords: Suction/injection, porous medium, convective flow, MHD, Perturbation.**Nomenclatures**

| | |
|-------------|---|
| g | is acceleration due to gravity |
| T' | is fluid temperature |
| C' | is species concentration |
| B_0 | Magnetic field |
| D_m | Chemical molecular diffusivity |
| R_m | Chemical reaction parameter |
| C_p | Specific heat at constant pressure |
| k | Thermal conductivity |
| G_r | Grashof number |
| P_r | Prandtl number |
| M | Magnetic field parameter |
| T'_∞ | Temperature of the fluid at infinity (K) |
| T'_w | Temperature of the fluid far away of the fluid from the plate (K) |
| * | Dimensionless properties |
| ω | Conditions on the wall |
| ∞ | Free stream conditions |

1.0 INTRODUCTION

Magneto hydrodynamic (MHD) is applied to study the stellar and solar structure, interstellar matter, radio propagation etc. The ionized gas or plasma can be made to interact with the magnetic field and can frequently alter heat transfer on the bounding surface. The investigation for the underground

water resources, seepage of water in river beds, the filtration and water purification processes in chemical engineering, one needs the knowledge of the fluid flow through porous medium. The porous medium is in fact a non-homogeneous medium but for the sake of analysis, it may be possible to replace it with a homogeneous fluid saturated medium which has dynamical properties equal to those of non-homogeneous continuum (Swapna *et al.*, 2017). Ojmeri *et al.* (2023) researched on MHD free convection flow of a Casson fluid that is electrically conductive, affected by thermal radiation effect in a vertical porous channel. The resulting solutions of the leading differential equations describing the velocity and temperature flows characteristics are obtained analytically using regular perturbation method. Omokhuale and Dange (2023) revealed the impact of heat absorption on unsteady MHD convective Jeffery flow of a viscous, electrically conducting and incompressible fluid. Reddy *et al.* (2019) examined the effects of chemical reaction on a transient MHD mixed convection flow with mass transfer past an impulsively fixed infinite vertical plate under the

influence of a transverse magnetic field have been presented. The medium is considered to be non-scattering and the fluid to be non-gray having emitting-absorbing and optically thick radiation limit properties.

Suction/injection method was first introduced one of the means for preventing or delaying boundary layer separation. Suction/injection is one of the methods of boundary layer control, which have the aim of reducing losses of energy in channels. The study of injection/suction on the boundary layer control played an important role in the field of aerodynamics and space sciences. By controlling the flow as such, fuel consumption might be decreased by 30%; a considerable reduction in pollutant emission is achieved. In mass transfer cooling, suction/injection of a fluid through the bounding significantly change the flow field and as a result, affect the heat transfer rate from the plate.

Injection is defined as the administering a fluid in to a system as in the case of blood transfusion while in the case of suction is defined as the exclusion of fluid from a system. If the two runs at the same time, then the opposite sides of the plates are porous which allow the fluid to move in and out. Suction/injection of fluid channels has gained a special concerned due to its paramount applications of different field such as Science, engineering, petroleum drilling industries and food processing industries etc (Usman *et al.*, 2023).

The flow problem through a porous media finds many applications in geophysical and geothermal applications. These applications include spreading of chemical pollutants in water-saturated soil, disposal of nuclear wastes, migration of moisture in fibrous insulation, clean-up of refineries and extraction of geothermal energy, gas assisted injection molding, die filling processes The effect of heat source on mixed convection flow of a viscous incompressible and electrically conducting fluid along an infinite vertical plate with magnetic field has been studied by Investigated the influence of variable thermal conductivity and heat (Zayyanu *et al.*, 2022).

A porous medium is a substance that contains pores, or spaces between solid materials through

which liquid or gas can pass. Yale and Uwanta (2019) studied Soret effect on magneto hydrodynamics flow of a viscous, incompressible, electrically conducting fluid past an infinite vertical plate with heat and mass transfer has been studied. The governing equations are solved analytically using perturbation technique.

Convection is the process of heat transfer by the bulk movement of molecules within fluids such as gases and liquids. The initial heat transfer between the object and the fluid takes place through conduction, but the bulk heat transfer happens due to the motion of the fluid (Takhar, 2007). Suction This is a production of partial vacuum by the removal of air in order to force fluid into a vacant space procedure adhesion (Chamkha, 2004). Injection is used to control the fluid flow in the channel, and an exothermic chemical reaction of Arrhenius kinetic is considered (Mitra and Das, 2009).

Heat and mass transfer in porous medium are known to have applications in industrial, chemical engineering, nuclear reactors, geophysical and in petroleum industries. Recent development in binary flow of mixtures and the determination of molecular weights separation of isotopes, food processing and filtration process lead to increased investigations understanding such flow (Dangana and Amos, 2020).

2.0 MATHEMATICAL FORMULATION

We considered an unsteady flow of an incompressible, viscous, electrically conducting and heat-absorbing fluid through a semi-infinite vertical plate. A uniform transverse magnetic field of magnitude B_0 is applied in the presence of temperature and concentration buoyancy effects in the direction of the y^* – axis. The transversely applied magnetic field and magnetic Reynolds number is assumed to be very small so that the induced magnetic field is negligible. We assumed that there is no applied voltage which implies the absence of an electric field. The physical variables are independent of the x^* – axis. We further assume that there exist a homogeneous first order chemical reaction between the diffusing species and the fluid. The presence of variable injection/suction is considered.

The governing equations under the usual Boussinesq's and boundary layer approximations are:

Continuity equation

$$\frac{\partial v^*}{\partial y^*} = 0 \tag{1}$$

Momentum equation

$$\frac{\partial u^*}{\partial t^*} + v^* \frac{\partial u^*}{\partial y^*} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + g\beta(T^* - T_\infty^*) + g\beta^*(C^* - C_\infty^*) + v \frac{\partial^2 u^*}{\partial y^{*2}} - \frac{v}{K^*} u^* - \frac{\sigma B_0^2}{\rho} u^* \tag{2}$$

Energy equation

$$\frac{\partial T^*}{\partial t^*} + v^* \frac{\partial T^*}{\partial y^*} = \frac{k}{\rho c_p} \frac{\partial^2 T^*}{\partial y^{*2}} - \frac{Q_0}{\rho c_p} (T^* - T_\infty^*) \tag{3}$$

Mass Equation

$$\frac{\partial C^*}{\partial t^*} + v^* \frac{\partial C^*}{\partial y^*} = D \frac{\partial^2 C^*}{\partial y^{*2}} - K_c^* (C^* - C_\infty^*) \tag{4}$$

The boundary conditions associated with the models are given as (The boundary conditions it is in dimensional or non-dimensional form)

$$u^* = 0, T^* = T_w^* + \epsilon(T_w^* - T_\infty^*)e^{int}, C^* = C_w^* + \epsilon(C_w^* - C_\infty^*)e^{int} \text{ at } y^* = 0 \tag{5}$$

$$u^* \rightarrow 0, T^* \rightarrow 0, C^* \rightarrow 0 \text{ as } y^* \rightarrow \infty$$

From equation (1), it is clear that the injection velocity at the plate is either a constant and or a function of time. Hence, the injection velocity normal to the plate is assumed in the form:

$$\text{where } v^* = v_0(1 + B\epsilon e^{int}) \text{ and } -\frac{1}{\rho} \frac{\partial p}{\partial x} = \lambda e^{int} \tag{6}$$

where B is a real positive constant ϵ and ϵB is small values less than unity, and v_0 is scale of injection velocity which is non zero positive constant. The positive sign indicates that injection is towards the plate.

In order to write the governing equations and the boundary condition in dimensional following non-dimensional quantities in equation (7) are introduced into equations (1) to (4).

$$y = \frac{v_0 y^*}{v}, u = \frac{u^*}{U_0}, t = \frac{t^* v_0^2}{v}, \eta = \frac{T^* - T_\infty^*}{T_w^* - T_\infty^*}, \chi = \frac{C^* - C_\infty^*}{C_w^* - C_\infty^*} \tag{7}$$

Therefore, equations (1) to (4) are reduced to following dimensional forms:

$$\frac{\partial u}{\partial y} + (1 + B\epsilon e^{int}) \frac{\partial u}{\partial y} = \lambda e^{int} Gr \eta + Gm \chi + \frac{\partial^2 u}{\partial y^2} - \frac{u}{k} - Mu \tag{8}$$

$$\frac{\partial \eta}{\partial t} + (1 + B\epsilon e^{int}) \frac{\partial \eta}{\partial y} = \frac{1}{Pr} \frac{\partial^2 \eta}{\partial y^2} - S\eta + \chi \frac{\partial^2 C}{\partial y^2} \tag{9}$$

$$-\frac{\partial \chi}{\partial t} + (1 + B\epsilon e^{int}) \frac{\partial \chi}{\partial y} = \frac{1}{Sc} \frac{\partial^2 \chi}{\partial y^2} - N\chi \tag{10}$$

The corresponding boundary conditions (5) in dimensionless form are:

$$u = 0, \eta = 1 + \epsilon e^{int}, \chi = 1 + \epsilon e^{int} \text{ at } y = 0 \tag{11}$$

$$u \rightarrow 0, \eta \rightarrow 0, \chi \rightarrow 0 \text{ as } y \rightarrow \infty$$

3.0 SOLUTION OF THE PROBLEM

The equations (8) to (10) are non-linear and coupled PDEs. In order to obtain solutions for the velocity, temperature and concentration of the fluid, we assume solution of the form:

$$u(y, t) = u_0(y) + u_1(y)\epsilon e^{int} \tag{12}$$

$$\eta(y, t) = \eta_0(y) + \eta_1(y)\epsilon e^{int} \tag{13}$$

$$\chi(y, t) = \chi_0(y) + \chi_1(y)\epsilon e^{int} \tag{14}$$

Substituting the Equations (12) to (14) in to equations (8) to (10).

$$\frac{d^2 u_0}{dy^2} - \frac{du_0}{dy} + a_5 u_0 = Gr\eta_0 - Gm \chi_0 \tag{15}$$

$$\frac{d^2 u_0}{dy^2} - \frac{du_0}{dy} + (\lambda - \frac{1}{k} - m - in)u_1 = \frac{Bdu_0}{dy} Gr\eta_1 - Gm\chi_1 \tag{16}$$

$$\frac{d^2 \eta_0}{dy^2} - \frac{Prd^2 \eta_0}{dy^2} - SPr\eta_0 = 0 \tag{17}$$

$$\frac{d^2 \eta_1}{dy^2} - \frac{Prd\eta_0}{dy} - (S + in)\eta_1 = \frac{Bd\eta_0}{dy} \tag{18}$$

$$\frac{d^2 \chi_0}{dy^2} - Sc \frac{d\chi_0}{dy} - NSc\chi_0 = 0 \tag{19}$$

$$\frac{d^2 \chi_1}{dy^2} - BSc \frac{d\chi_1}{dy} - (NSc + in)\chi_1 = \frac{Bd\chi_0}{dy} \tag{20}$$

The boundary condition can be written as :

When $y \rightarrow 0, u_0 \rightarrow 0$, When $y \rightarrow \infty, u_1 \rightarrow 0$,
 When $y = 0, \eta_I = 1$, When $y \rightarrow \infty, \eta_I \rightarrow 0$,
 When $y = 0, \chi_o = 1$, When $y \rightarrow \infty, \chi_o = 0$

$$(21)$$

Solving equations (15) to (20) subject to boundary conditions (21), we obtain the velocity, temperature and concentration distributions as:

Velocity Distribution

$$u(y, t) = A_{12}e^{-d_{10}y}A_{13}e^{-d_6y} + A_{14}e^{-d_6y} \quad (22)$$

Temperature Distribution

$$\eta(y, t) = e^{-d_8y} + (A_9e^{-d_8y} + A_{10}e^{-d_6y})\epsilon e^{int} \quad (23)$$

Concentration Distribution

$$\chi(y, t) = e^{-d_2y} + (A_4e^{-d_2y} + A_5e^{-d_2y})\epsilon e^{int} \quad (24)$$

The skin-friction, Nusselt number and Sherwood numbers are important physical parameters for this type of boundary layers flow.

3.1 Skin friction

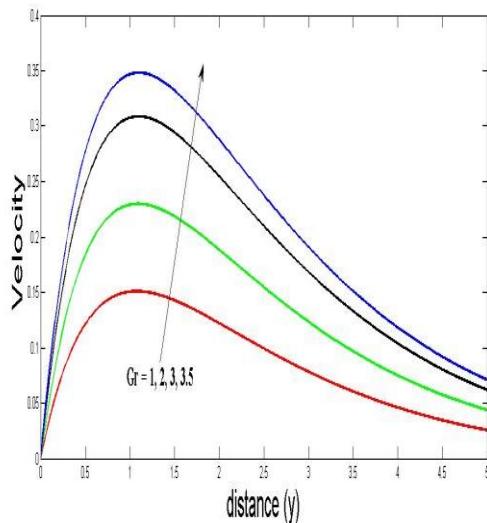
The skin-friction at the plate is given by:

$$\left. \frac{du}{dy} \right|_{y=0} = d_{10} A_{12} - d_6 A_{13} + d_2 A_{14} + [-d_{12} A_{16} - d_{10} A_{17} - d_2 A_{18} - d_2 A_{19} - d_8 A_{20} - d_4 A_{21}] \epsilon e^{int}$$

3.2 Nusselt numbers

The rate of heat transfer is given by:

$$\left. \frac{d\eta}{dy} \right|_{y=0} = -d_6 + [-d_8 A_9 - d_6 A_{10}] \epsilon e^{int}$$



3.3 Sherwood Number

The rate of mass transfer is:

$$\left. \frac{d\chi}{dy} \right|_{y=0} = -d_2 + (-d_4 A_4 - d_2 A_5) \epsilon e^{int}$$

4.0 RESULTS AND DISCUSSION

To gain more insight on the effects of different parameters associated with flow, the numerical results of the fluid velocity, temperature, concentration, Skin-friction, Nusselt and Sherwood numbers are computed. Figures 4.1 and 4.2 show the effects of thermal and mass Grashof numbers on the velocity profiles. It is seen that there is rise in the velocity because of the enhancement of thermal buoyancy force. Also, the peak value of the velocity is higher near the porous plate and decays smoothly for free steam velocity. It is further observed that the fluid velocity increases and the peak value is more distinctive due to significance in species buoyancy force. Thus, the velocity distribution attains a distinctive maximum value in the vicinity of the plate and then reduces properly to approach free stream value. The effect of Magnetic parameter (Hartmann number) on the velocity profiles is as illustrated in Figure 4.3. It is observed that the velocity of the fluid falls with increasing magnetic parameters. This is due to the fact that the presence of magnetic field in an electrically conducting fluid sets in a force called Lorentz force which acts against the flow if the magnetic field is applied in the normal direction as reflected in the problem. This type of resistive force has a tendency to slow down the flow field.

Figure 4.1: Effect of Gr on the velocity profiles.

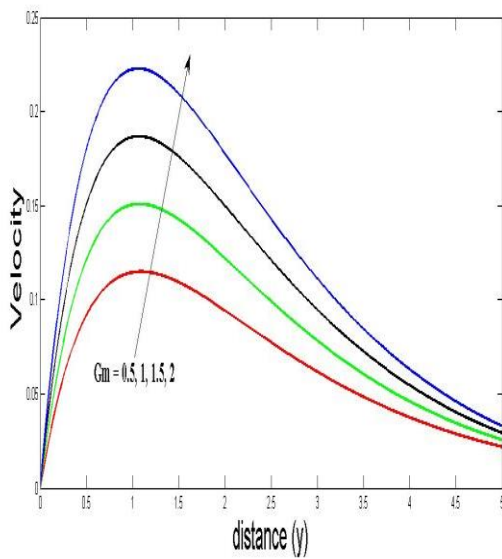


Figure 4. 2: Effect of Gc on the velocity profiles.

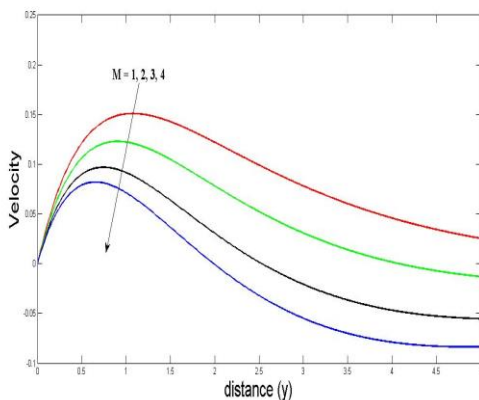


Figure 4. 3: Velocity profiles for different values of M .

It is clear from Figure 4.4 that higher values of injection parameter causes rise in the fluid velocity. It indicates that there is an enhancement in the fluid velocity as injection progresses. Figure 4. 5 depicts the effect of Prandtl number on the velocity profiles. It is shown that the velocity becomes lower as the values of Prandtl number increases. Physically, this happens since fluid with high Prandtl number have greater viscosity, which makes the fluid thick and then move slowly. The effect of Schmidt number Sc for different values on the velocity profiles are displayed in Figure 4.6. It is noticed that increase in Sc leads decrease in the velocity throughout the momentum boundary layer. However, a distinct velocity overshoot exists near

the plate, and thereafter the profiles fall to zero at the edge of the boundary layer.

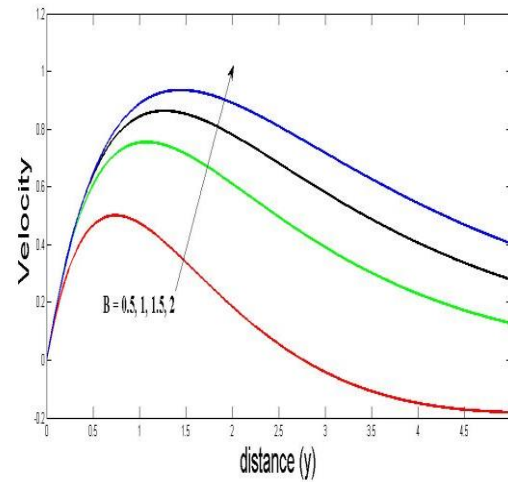


Figure 4. 4: Effect of B on the velocity profiles.

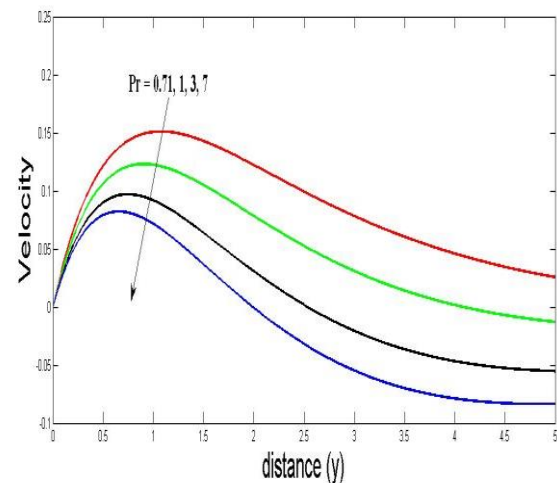


Figure 4.5: Effect of Pr on the velocity profiles.

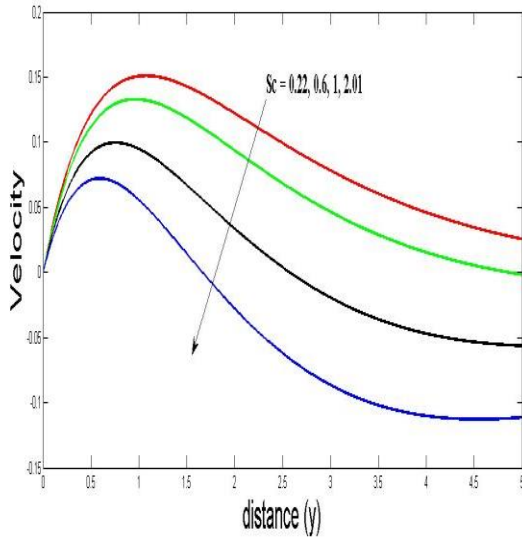


Figure 4.6: Effect of Sc on the velocity profiles.

Figure 4.7 presents the effect of Prandtl number on the temperature profiles. It is seen that the temperature reduces as Prandtl number is becomes higher. The effect of injection parameter on the velocity profiles in the boundary layer is shown in Figure 4.8. It is observed that increase in the injection parameter results in higher velocity within the thermal boundary layer. In Figure 4.9, the effect of heat generation parameter for different values on the temperature profiles are plotted. It is found that its increase reduces the thermal boundary layer.

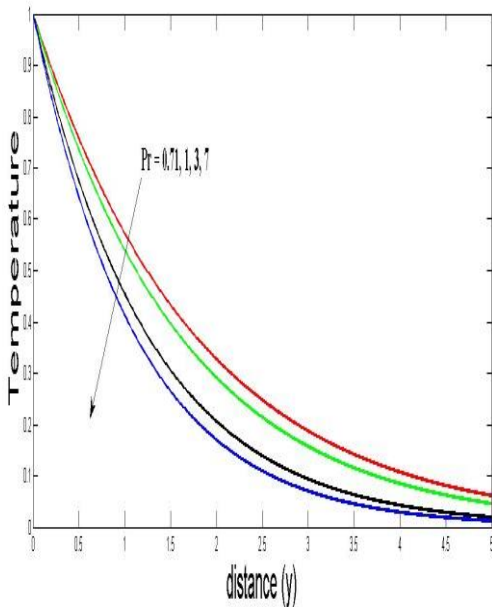


Figure 4.7: Effect of Pr on the temperature profiles.

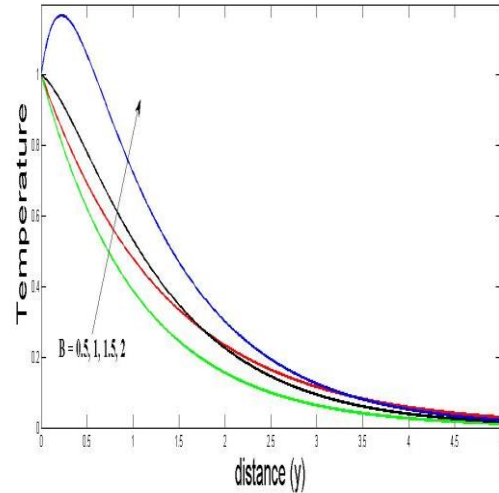


Figure 4.8: Effect of B on the temperature profiles.

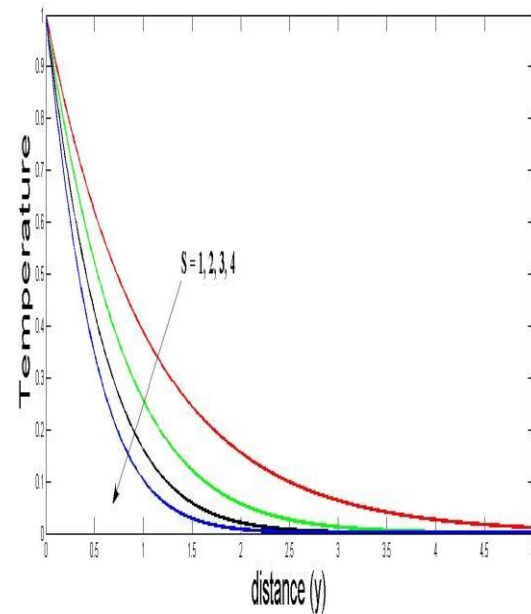


Figure 4.9: Effect of S on the temperature profiles.

Figures 4.10 and 4.11 give the concentration field due to variation in Schmidt number and chemical reaction parameter respectively. It is seen that the concentration of the fluid become lower as the Schmidt number rises. Physically, this holds because of the fact that increase in Sc means decrease of molecular diffusivity, which causes decrease of concentration boundary layer while a similar trend is noticed when chemical reaction parameter is significant. Hence, the concentration of species is smaller for higher values of Sc . Figure

4. 12 shows the concentration profiles for different values of injection. It is depicted that the concentration increases as injection becomes higher.

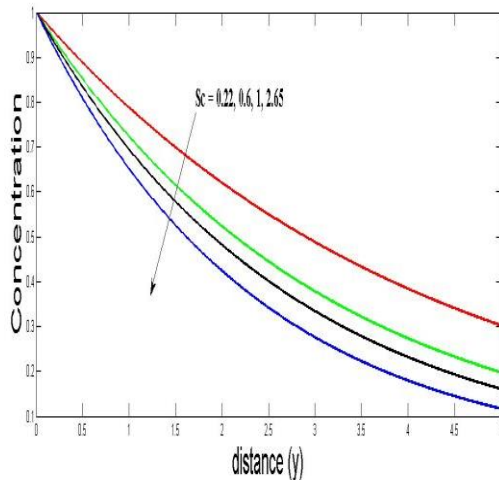


Figure 4. 11: Effect of Rc on the concentration profiles.

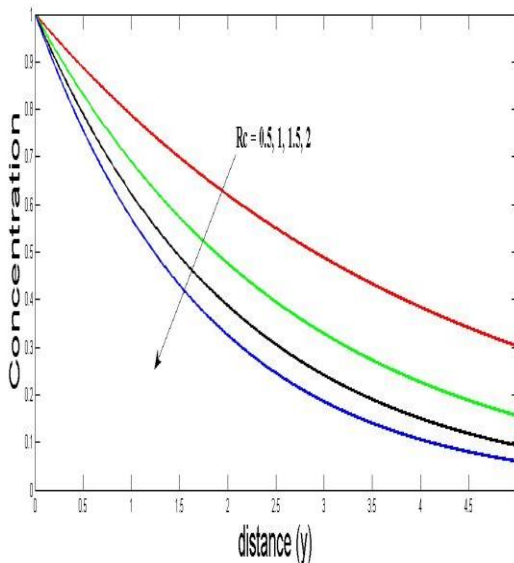


Figure 4. 10: Effect of Sc on the concentration profiles.

5.0 CONCLUSION

Time dependent injection/suction and heat source/sink effects on free convective flow in a porous medium have been studied. The coupled PDEs are solved by perturbation technique. Numerical experiment is performed using MATLAB. Results are shown as Figures and discussed for physical parameters associated with

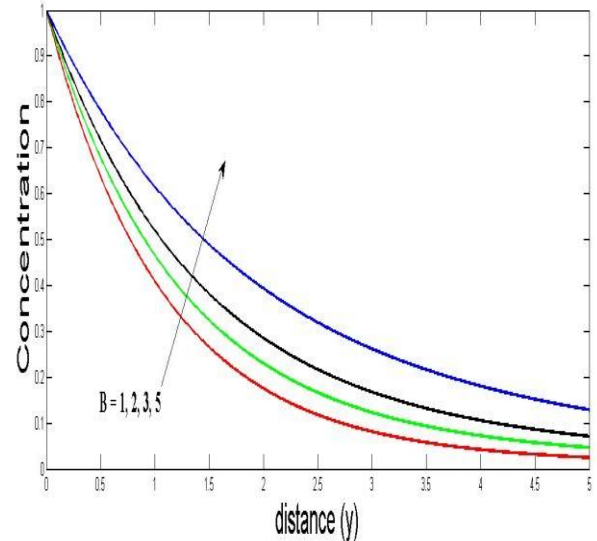


Figure 4. 12: Effect of B on the concentration profiles.

the flow of the fluid. The following conclusion were drawn from the study:

- i. Velocity of the fluid becomes higher increasing in mass and thermal Grashof numbers.
- ii. The momentum, mass and thermal boundary layers increases as injection becomes significant while a opposite trend is seen for increased values of suction parameter.
- iii. The velocity of the fluid reduces for both constant and time dependent pressure parameter.
- iv. The momentum and thermal boundary layers increase due to significance of heat source parameter an opposite trend was noticed when the heat source parameter is higher.
- v. The temperature of the fluid reduces as Prandtl number is increased.
- vi. The concentration profile falls as Schmidt number become significant, chemical reaction and the reverse trend is seen as injection parameter is increased.

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