

**INTERNATIONAL JOURNAL OF SCIENCE FOR GLOBAL SUSTAINABILITY****On the Application of Geometric Brownian Motion Model to Stock Price Process on the Nigerian Stock Market****Imoni O. Sunday and Muhammad S. Sulaiman**

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**ABSTRACT**

This Study presents the application of Geometric Brownian Motion (GBM) model for the prediction of stock prices in the Nigeria Stock Market. We analyzed and examined the appropriateness of the GBM model in predicting stock prices on the Nigeria Stock market. Fluctuations of stock prices on daily basis make them more volatile and difficult for financial managers and investors to predict future stock prices. This paper, therefore, aims to give the importance of the application of GBM to stock prices and help financial managers and investors in forecasting future stock prices for a short-term investment. The sample for this study was based on listed stock quoted on the Nigerian stock exchange (NSE). Daily stock prices data of Nestle Foods, First Bank, Associated Bus Company, MRS Oil, Dangote Cement and Transnational Corporation were obtained from updates of the official website of the NSE over a period of one month. It was observed that within a short period of time, the GBM is effective in precisely predicting the daily stock prices of stocks analyzed and examined.

**Keywords:** geometric brownian motion, forecasting, stock prices, volatility, stock market

**1.0 INTRODUCTION**

A stock market is a place where shares of public listed companies are traded. Stocks, also known as equities, represent fractional ownership in a company and the stock is a place where investors can buy and sell ownership of such investible assets. The stock market plays a vital role in the growth of the industry and commerce of the country that eventually affects the economy of the country to a great extent. This is the reason why the government, industry and even the central bank of the country keep a closed watch in the happenings of the stock market.

The stock markets is important from both the industry's point of view as well as the investor's point of view.

Changes of share prices on daily basis make them more volatile and difficult to predict. When purchasing a stock, it does not guarantee anything in return. Thus, it makes stocks risky in investment but investors can gain high return.

Wrong decision in choosing the counters may end up in capital loss (Abidin and Jaffar, 2012).

This paper provides a basic guide for investors and other financial managers to predict future share prices using GBM model. The model can predict share prices in a short period of time by taking into account the importance elements of the share process (Abidin and Jaffar, 2012). Investment in short period of time is the time awaited by every investor to earn profit immediately.

Financial managers and investors are interested in simulating the price of stock, options and derivatives in order to make important investment and financial decisions. Stimulating the prices of stock means generating price paths that a stock may follow in the future (See, Krishna, 2016) for more details. We talk about simulating stock prices because future stock prices are uncertain (called stochastic) but we believed that they follow at least approximately, a set of rules that can be derived

from historical data and our knowledge of stock prices (Sengupta *et al.*, 2004)

In this study, we focus on the GBM model for predicting stock prices and to determine if the GBM model is effective in predicting stock prices in the Nigerian stock market.

### 1.1 Literature Review

Brownian motion is often used to explain the movement of time series variables in corporate finance and in the movement of asset prices. Brownian motion dates back to the nineteenth century when it was discovered by biologist Robert Brown examining pollen particles floating in water under the microscope, (Erogenous, 2005). Brown observed that the pollen particles exhibited a jittery motion and concluded that the particles were 'alive'. This hypothesis was later confirmed by Albert Einstein in 1905 who observed that under the right conditions, the molecules of water moved at random. A common assumption for stock markets is that they follow Brownian motion, where asset prices are constantly changing often by random amounts, (Ermogenous, 2005). This concept has led to the development of a number of models based on radically different theories. Two common approaches to predicting stock prices are those based on the theory of technical analysis and those based on the theory of fundamental analysis. Technical theorists assume that history repeats itself, that is, past patterns of price behavior tend to recur in future. The fundamental analysis approach assumes that at any point in time an individual security has an intrinsic value that depends on the earning potential of the security, meaning some stocks are overpriced or underpriced (see Fama and French, 1995). Many believe in an entirely different approach, the theory that stock market prices exhibit random walk. The random walk theory is the idea that stocks take a random and unpredictable path, making it near impossible to outperform the market without assuming additional risk. This theory casts serious doubts on the other methods of describing and predicting stock price behavior.

The GBM model incorporates this idea of random walks in stock prices its uncertain components, along with the idea that stocks maintain price trends over time as the certain component. Brewer *et al.*, (2012) described the uncertain component to the GBM model as the product of the stock's volatility and a stochastic process called Wiener process, which incorporates random volatility and a time interval.

Sengupta (2004) states that for GBM model to be effective, one must imply that:

- (i) The company is a going concern, and its stock prices are continuous in time and value.
- (ii) Stocks follow a Markov process, meaning only the current stock price is relevant for predicting future prices.
- (iii) The proportional return of a stock is log-normally distributed.
- (iv) The continuously compounded return for a stock is normally distributed.

Each of these assumptions has an effect on the GBM model and its inputs. GBM has two components; a certain component and an uncertain component. The certain component represents the return that the stock will earn over a short period of time, also referred to as the drift of the stock. The uncertain component is a stochastic process including the stocks volatility and an element of random volatility (see, Sengupta (2004)). Brewer, Feng and Kwan (2012), show that only the volatility parameter is present in the Black-Scholes (BS) model, but the drift parameter is not, as the BS model is derived based on the idea of arbitrage-free pricing. For Brownian motion simulations, both the drift and volatility parameters are required, and a higher drift value tends to result in higher simulated prices over the period being analyzed (Brewer, Feng and Kwan, 2012). Although the GBM process is well-supported, there is a growing amount of literature that focus on testing the validity of the model and accuracy of forecasts using Brownian motion.

For example, Abidin and Jaffar (2014) used GBM to forecast future closing prices of small-sized companies in Bursa Malaysia. The study focuses on small-sized companies because the asset prices are lower and more affordable for individual investors. The study looks into the accuracy of forecasts made using the GBM model over different horizons, and also at the time horizon needed for data inputs into the model, that is, past stock prices. Marathe and Ryan (2005) discussed the process for checking whether a given time series follows the BGM process. They also look at methods to remove seasonal variation from a time series, which they claim is important because the GBM process does not include cyclical or seasonal effects. They found that of the four industries they studied, the time series for usage of established services met the criteria for a GBM process, while the data from growth of emergent services did not. Also see (Krishna and Vaughan, 2016) for more details.

Despite several literatures surrounding the pricing of securities in corporate finance, however there is still a lot of debate as to which method is the most reliable. Financial managers and investors are interested in simulating the price of stock, options, and derivatives in order to make important investment and finance decisions.

Several researchers have used other methods to forecast stock prices. For example, some of these models are summarized by Granger (1992), with a particular emphasis on non-linear models. Higgins (2011) presented a simple model to forecast stock prices using analyst earnings forecasts based on the residual income model (RIM). He showed how to implement the RIM and explains how it can be adjusted for auto-correlation to improve forecast accuracy. The RIM uses a combination of fundamental accounting data and mechanical analysis of trends in time series data to derive a valuation of the firm. Hadavandi *et al.* (2010) presented an integrated approach based on genetic fuzzy systems (GFS) and artificial

neural networks (ANN) for constructing a stock price forecasting expert system. They apply the model to stock price data gathered from IT and Airline sectors, and compare the outcomes with previous stock price forecasting methods using mean absolute percentage error (MAPE). The results they obtained showed that the proposed approach outperforms all previous methods, and so it can be considered a suitable tool for stock price forecasting. However, a drawback to the MAPE approach is that it can only be used to predict the next day closing prices, making it less relevant to managers making strategic decisions

According to Hsu, Liu, Yeh and Hung (2009), a combination of grey model (GM), Fourier series and Markov state transition matrices to produce a new integrated prediction method called the Markov-Fourier grey model (MFGM). The hybrid model was used to predict the turning time of Taiwan weighted stock index (TAIEX) in order to improve forecasting accuracy. According to Hsu *et al.* (2009). MFGM method can predict accurately but it is only suitable for long-term operation. Hassan and Nath (2005), developed high-order fuzzy time-series model. Chen *et al.* (2008), proposed moving average auto-regressive exogenous (ARX) with combination of grey system (GS) and rough sets (RS), Huang and Jane (2009) constructed the clustering-genetic fuzzy system (CGFS), Hadavandi *et al.* (2010) presented an integrated approach based on genetic fuzzy systems (GFS) and artificial neural networks (ANN) for constructing a stock price forecasting expert system. They apply their model to stock price data gathered from IT and Airline sectors, and compare the outcomes with previous stock price forecasting methods using mean absolute percentage error (MAPE). The results they obtained showed that the proposed approach outperforms all previous methods, and so it can be considered a suitable tool for stock price forecasting. However, a drawback to the MAPE approach is that it can only be used to predict the next day closing prices,

making it less relevant to financial managers making strategic decisions.

**2.0 THE GEOMETRIC BROWNIAN MOTION MODEL**

**Definition:** A Stochastic (random) process  $S_t$  is said to follow a GBM, if it satisfies the following stochastic differential equation (SDE):

$$dS_t = \mu S_t dt + \sigma S_t dW_t \tag{2.1}$$

where  $W_t$  is a Wiener process or Brownian motion,  $dS_t$  is the change in the stock price in an infinitesimal time,  $dt$  is an infinitesimal change in time,  $\mu$  is the percentage drift and  $\sigma$  is the percentage volatility are constant, (Dampney, 2017).

The price of stock in nearest possible time  $dt$  can be divided into two parts: the predictable part which is the expected rate of return on the stock at the  $dt$ ,  $\mu S_t dt$  and a stochastic and unexpected part which is as a result of the random changes in the stock price over time  $dt$ ,  $\sigma S_t dW_t$ , where  $dW_t$  is Brownian motion or Wiener process.

Solving the stochastic differential equation (2.1) using method of separation of variables, we obtain

$$\int \frac{dS_t}{S_t} dt = \int (\mu dt + \sigma dW_t) dt \tag{2.2}$$

$$\ln\left(\frac{dS_t}{S_t}\right) = \left(\mu - \frac{\sigma^2}{2}\right)t + \sigma W_t \tag{2.3}$$

$$S_t = S_0 \exp\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma W_t \tag{2.4}$$

$$E(S_t) = S_0 \exp\left(\mu + \frac{\sigma^2}{2}\right)t \tag{2.5}$$

$$\text{var}(s_t) = S_0^2 e^{(2\mu + \sigma^2)t} (e^{\sigma^2 t} - 1) \tag{2.6}$$

where equations (2.5) and (2.6) represent the expected and variance of the solution (2.4) at any time  $t$ . Also, using (2.2), the variable  $\ln s_t : N(\ln s_0 + (\mu - \frac{\sigma^2}{2})t, \sigma^2 t)$ . Thus, at 95%

confidence interval, the confidence interval of  $s_t$  is given as

$$e^{\ln s_0 (\mu - \frac{\sigma^2}{2})t - 1.96(\sigma\sqrt{t})} \leq s_t \leq e^{\ln s_0 (\mu - \frac{\sigma^2}{2})t + 1.96(\sigma\sqrt{t})} \tag{2.7}$$

Also, for arbitrary initial value  $s_0$  the differential equation (3.1) can be derived using Ito calculus. Applying the Ito's formula gives

$$\begin{aligned} d(\ln S_t) &= (\ln S_t)' dS_t + \frac{1}{2} (\ln S_t)'' dS_t dS_t \\ &= \frac{dS_t}{S_t} - \frac{1}{2} \frac{1}{S_t^2} dS_t dS_t \end{aligned} \tag{2.8}$$

where  $dS_t dS_t$  is the quadratic variation of the stochastic differential equation.

$$dS_t dS_t = \sigma^2 S_t^2 dt + 2\sigma S_t^2 \mu dW_t dt + \mu^2 S_t^2 dt^2 \tag{2.9}$$

When  $dt \rightarrow 0$ ,  $dt$  converges to 0 faster than  $dW_t^2 = O(dt)$ . Thus, the above can be simplified by

$$dS_t dS_t = \sigma^2 S_t^2 dt \tag{2.10}$$

Insert the value of  $dS_t$  in the above equation and simplify to obtain

$$\ln \frac{S_t}{S_0} = \left(\mu - \frac{\sigma^2}{2}\right)t + \sigma W_t \tag{2.11}$$

By taking the exponential and multiply both sides by  $S_0$  gives the solution (2.4)

**2.1 Forecast Measure and Data Analysis**

There are three measurements of forecasting model accuracy which involves time period,  $t$ . The measurements are number of period forecast,  $n$ , actual value at time period  $t$ ,  $A_t$ , and forecast value at time  $t$ ,  $F_t$  (see, Lawrence *et al.* 2009) for more details. The forecast accuracy of our model was carried out using the Mean Absolute Percentage Error (MAPE)

$$\text{given as MAPE} = \frac{1}{n} \sum_{t=1}^n \left| \frac{A_t - F_t}{A_t} \right| \tag{2.12}$$

This measures the deviation between the forecasted values and the actual stock values. Table 2.1 depicts a scale of judgment of forecast accuracy using MAPE. Table 2.2 shows actual daily stock value of Nestle Foods, First Bank, Associated Bus Company, MRS Oil, Dangote Cement and Transnational Corporation for a period of one month

MAPE (%)	Judgment of forecast accuracy
< 10%	Highly accurate
11% - 20%	Good forecast
21% - 50%	Reasonable forecast
>51%	Inaccurate forecast

Table 2.1: Scale of Judgment of Forecast Accuracy (Abidin and Jaffar, 2012)

Table 2.2: Daily Prices of Stocks

Date	Daily Prices of Stocks Data (Actual) in Naira (₦)					
	Nestle Foods	First Bank	Associated Bus Company	MRS Oil	Dangote Cement	Transnational Corporation
01-MAY-19	3.00	3.40	3.80	1.85	2.85	2.69
02-MAY-19	3.00	3.55	3.80	1.90	3.00	2.69
03-MAY-19	3.00	3.55	3.80	1.98	2.94	2.69
06-MAY-19	3.09	3.83	3.79	2.17	2.93	2.74
07-MAY-19	3.10	4.12	3.79	2.18	2.80	2.62
08-MAY-19	3.07	4.02	3.78	2.13	2.90	2.59
09-MAY-19	3.07	3.77	3.74	1.97	2.90	2.58
10-MAY-19	3.09	3.90	3.78	2.15	2.94	2.68
13-MAY-19	3.00	3.82	3.78	2.20	2.80	2.74
14-MAY-19	3.25	3.79	3.78	2.47	2.80	2.60
15-MAY-19	3.35	3.54	3.78	2.57	2.75	2.65
16-MAY-19	3.35	3.58	3.78	2.82	2.80	2.59
17-MAY-19	3.31	3.60	3.75	3.05	2.75	2.57
20-MAY-19	3.40	3.67	3.74	2.89	2.79	2.62
21-MAY-19	3.40	3.63	3.78	2.80	2.65	2.73
22-MAY-19	3.32	3.63	3.78	2.81	2.60	2.75
23-MAY-19	3.32	3.63	3.78	2.70	2.35	2.72
24-MAY-19	3.35	3.59	3.78	2.80	2.58	2.75
27-MAY-19	3.35	3.59	3.78	2.80	2.58	2.75
28-MAY-19	3.35	3.59	3.78	2.89	2.64	2.77
29-MAY-19	3.54	3.80	3.78	2.90	2.64	2.77
30-MAY-19	3.40	3.77	3.79	2.89	2.58	2.75
31-MAY-19	3.35	3.75	3.78	2.90	2.64	2.77

Source: Daily Updates of the Official Website of the Nigerian Stock Exchange ([www.nse.gov.ng](http://www.nse.gov.ng))

Apply equation (2.4) to predict the future stock values for Nestle Foods, First Bank, Associated Bus Company, MRS Oil, Dangote Cement and Transnational Corporation. Table 2.3 shows a

sample of predicting value for Dangote Cement. It is one month forecasting data compared with the actual prices.

Table 2.3: A sample of predicted prices vs. actual prices of Stocks for Dangote Cement

Date	Actual Values (N)	Predicted Values (N)
01-May-19	2.85	2.92
02-MAY-19	3.00	3.08
03-MAY-19	2.94	3.01
06-MAY-19	2.93	3.00
07-MAY-19	2.80	2.87
08-MAY-19	2.90	2.97
09-MAY-19	2.90	2.97

10-MAY-19	2.94	3.01
13-MAY-19	2.80	2.86
14-MAY-19	2.80	2.86
15-MAY-19	2.75	2.71
16-MAY-19	2.80	2.86
17-MAY-19	2.75	2.81
20-MAY-19	2.79	2.85
21-MAY-19	2.65	2.70
22-MAY-19	2.60	2.65
23-MAY-19	2.35	2.40
24-MAY-19	2.58	2,65
27-MAY-19	2.58	2.63
28-MAY-19	2.64	2.69
29-MAY-19	2.64	2.69
30-MAY-19	2.58	2,63
31-MAY-19	2.64	2.69

We now measure the accuracy of the forecast model by calculating the MAPE values using equation (2.8). Table 3.4 illustrates the MAPE

values of the listed companies under consideration.

**Table 2.4: A sample of MAPE values for the selected companies**

Listed Company	MAPE VALUE (%)
Nestle Foods	8.65
First Bank	0.53
Associated Bus Company	0.53
MRS Oil	8.40
Dangote Cement	2.13
Transnational corporation	5.19

From Table 2.4, it is observed that all the MAPE values are less than 10%, it implies that the GBM model is a highly accurate model for

forecasting these stock prices on the Nigeria stock market for the one month investment.

**Test for the Normality Assumption**

A further test is carried out using the Shapiro-Wilk test of normality to the daily stock prices under the following hypotheses:

H<sub>0</sub>: the distribution is normal

H<sub>1</sub>: the distribution is not normal

The approach used in the Shapiro-Wilk test for normality is as follows:

- (i) Rearrange the data in ascending order so that  $x_1 \leq x_2, \dots, x_n$

$$s = \sum_{i=1}^n (x_i - \bar{x})^2$$

- (ii) Calculate

$$b = \sum_{i=1}^m a_i (x_{n+1-i} - x_i)$$

- (iii) Calculate  $\frac{b}{s}$ , taking the  $a_i$ 's (based on the value of n) from the

Shapiro-Wilk tables, where  $m = \frac{n}{2}$  if n is

even and  $m = \frac{n-1}{2}$  if n is odd (n is the data

size).

- (iv) Calculate the test Statistic  $W = \frac{b^2}{s}$ . Find

the value in the Shapiro-Wilk tables (for a given value of n) that is closed to

$W$ , interpolate if necessary. This is the p-value for the test. If the test statistic  $W$  is smaller than the critical threshold, the assumption of normal distribution has to be rejected.

Table 2.5 shows the calculated values using the Shapiro-Wilk t test Statistic  $W = \frac{b^2}{s}$  of the listed companies under consideration.

**Table: 2.5 Values of Shapiro-Wilk Test Statistic**

Listed Company	$W = \frac{b^2}{s}$
Nestle Foods	0.8594781897
First Bank	0.6196199
Associated Bus Company	0.730
MRS Oil	0.847
Dangote Cement	0.846
Transnational corporation	0.6196199

From Table 2.5, all the calculated values are less than the values from Shapiro Wilk Tables (Shapiro and Wilk, 1965) and the p-value for the test is obtained as 0.914. Thus, since calculated values are less than the values from Shapiro Wilk Tables, we do not reject  $H_0$  and concluded that the samples are normally and independently distributed. Also since the p-value is greater than the level of significance (0.005), we arrive at the same conclusion

**3.0 CONCLUSION**

A review and application of the Geometric Brown Motion (GBM) model to some listed stock quoted on the Nigerian stock exchange (NSE) have been studied. Daily stock prices data of Nestle Foods, First Bank, Associated Bus Company, MRS Oil, Dangote Cement and Transnational Corporation were obtained from updates of the official website of the NSE over a period of one month. The study find out that, the model is appropriate for forecasting stock prices on the Nigeria stock market for the short time investment at least a maximum of one month investment.

GBM gives a good opportunity to decide new and gain profit after one month of investment. Also, GBM is highly accurate model, proven in Table 3.4, the most MAPE value are below 10%. The calculation of GBM is much easier and do not need a lot of data to forecast the future closing prices. Although, GBM model has some demerits but it proven that it gives the accurate value to the actual prices. Therefore, GBM is the best model used to forecast the future closing for at least a maximum of one

month investment. The Shapiro-Wilk test indicated that all the listed stocks fit the geometric Brownian motion (GBM) model.

**4.0 RECOMMENDATION**

As a recommendation, the model should be used to forecast daily stocks prices over short intervals as that is likely to give a more accurate forecast. Also, the model should be used to compare different sample sizes to further check its accuracy with respect to time.

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