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The Effectiveness of Variable Properties on Thermophoresis Particle Deposition at The Levels of 1st and 4th Orders of Chemical Reaction of Buoyancy-driven Flows of Second Grade Fluid

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ABSTRACT

This article presents the analytic solution to a steady, incompressible, free convective flow of an electrically conducting second grade fluid past a vertical surface with variable properties namely: variable viscosity; variable thermal conductivity and variable concentration diffusivity in the presence of thermophoresis, chemical reaction and convective boundary condition. The impact of different orders of chemical reaction on thermophoresis and the transport process of flow, heat and mass transfer in the boundary layer for assisting and opposing flow cases are properly examined and discussed. The governing equations associated with the fluid model are transformed and parameterized to a system of coupled nonlinear ordinary differential equations using similarity transformations. The resulting coupled ordinary differential equations (ODEs) were solved by adopting the Optimal Homotopy Analysis Method (OHAM). The results indicate that velocity and temperature distributions are decreasing functions of second grade parameter for both cases of assisting and opposing flows. Also, concentration distribution is a decreasing function of thermophoretic parameter for low and high orders of chemical reaction.

Keywords: Second grade fluid; Buoyancy; Chemical reaction; Optimal homotopy and thermophoresis

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1.0 Introduction

Convective heat transfer also known as Convection is defined as a mode of heat transfer by which heat energy is transferred between a surface and a moving fluid at different temperature. Convective heat transfer is regarded as dominant and special form of heat transfer in gases and liquids mainly because it involves the joint process of heat diffusion and heat transfer by bulk fluid flow. Convection can be categorized as either forced (assisted) convection or natural (free) convection depending on how the fluid flow is initiated. In forced convection, the flow is forced to flow over the surfaces by an external impetus such as fan, pump or the wind. On the other side, convection is termed natural or free if the fluid motion is generated by buoyancy forces that are induced by density difference due to the temperature variation in the fluid. During natural convection, an increase in temperature yields decline in density and hence, instigate fluid motion as a result of the pressures and forces when fluids of different densities are affected by gravity. For instance, during boiling process, when the fluid is heated from below, thermal expansion takes place, the hotter lower layer of the fluid becomes less dense and since it

is a true fact that colder fluid are more dense than hotter fluid, owing to buoyancy, the hotter, less dense part of the fluid moves up and the colder, denser fluid moves downward and replaces the hotter fluid. This process is repeated when the lower layer part also gets heat and moves upward to be replaced by the colder fluidand thus, convective cycle is set in. There are many applications attributed to free convection heat transfer. It influences the operating temperatures of power generating and electronic devices, It is important in establishing temperature distributions within the buildings and in determining heat losses or heat loads for heating, ventilating and air conditioning systems Frank *et al.*(2007)Grashof number is the dimensionless parameter used in the association with heat and mass transfer that governs the fluid flow in natural convection. The worth of this dimensionless number is that it corresponds to the ratio of the buoyancy force to the fluid viscous force acting on a fluid (see Theodera*et al.* (2007)for details). This dimensionless number is significant, the reason being that, buoyancy forces is what drives natural convection as the hot fluid moves upward and the cold fluid moves downward, and the viscous force tries to stop it. The study of free convection in a boundary layer flow is presented in the works of Barik *et al.* (2013), Singh and Madhab (2013).

The wide advancement of the non-linear fluids rheology has captivated the attention of many researchers attributable to their significance in industry and technology. Innon-Newtonian fluid, there is exhibition of non-linear relationship between the shear stress and the rate of deformation. The models are primarily characterized as rate, differential and integral type fluids. A major important class of non-Newtonian fluids is viscoelastic fluid, and the most commonly used simplest subclass of viscoelastic fluids which is differential type fluid is the Rivlin Ericksen fluid of grade two or second grade which was proposed by Rivlin and Ericksen (1955) and can predict the normal stress effects. The viscoelasticity of fluids result to an increase in order of differential equation(s) characterizing the flow. A few representative studies involving the flow of second grade fluid have been extensively reported in the works of Olanrewaju*et al.* (2016), Akinbola and Okoya (2015) and Olanrewaju and Abbas (2015)

Thermophoresis of particles is known as a mechanism of movement of small particles in the direction of decreasing thermal gradient. During the process, particles in hot environment migrate to a region where there is coldness. The effect of this phenomenon was first observed in 1870 by Tyndal (1870), when he observed that a particle free zone around a heated surface appeared in dusty air, and later in 1884, Aitken (1884) came up with a prove that the microscopic explanation to the effect was due to the heavier bombardment of the particle from the molecules on the hot region compared to cold region. In this phenomenon, the gas molecules migrating from the hot side of the particles have a greater velocity than those migrating from the cold side. The faster moving molecules collide with the particles more forcefully. This phenomenon has many applications in aerosol technology, deposition of silicon thin films and radioactive particle deposition in nuclear reactor safety simulations as reported by Hayat and Qasim (2010). Therefore, the velocity attained by the particle is referred to as thermophoretic velocity while the force experienced by the suspended particles due to the temperature gradient is referred to as thermophoretic force, and the direction of the force is opposite to the temperature gradient Stanford Shateyi (2013).

Chemical reaction is a process in which one or more substances are converted to one or more different substances. Chemical reactions as we know form an important part of technology and also day-to-day activities. Examples of processes that involve incorporating chemical reaction include burning fuels, making glass, brewing beer, making wine etc. In many instances in chemical reactions, the reaction rate depends on the concentration of the species itself and the order of chemical reaction with respect to a given substance (such as reactant, catalyst or product) is known to be index, or exponent, to which its concentration term in the rate equation is raised McNaught and Wilkinson (1997). The order of chemical reaction is known to depend on several factors and the simplest of which is the first order reaction, where the rate of reaction is directly proportional to the concentration of species Kundu *et al*. (2014). According to the study by Themelis (1995) it was stated that the majority of chemical reactions encountered in applications are first-order when a reaction rate depends on a single substance and the value of the exponent is one. Ferdows and Al-Mdallal (2012) presented effects of order of chemical reaction on a boundary layer flow with heat and mass transfer over a linearly stretching sheet. They concluded that the flow profiles increases with the increase of order of chemical reaction and decreases with the increase of Schmidt number and chemical reaction parameter. Studies involving heat and mass transfer in the presence of chemical reaction are also revealed in Kasmani *et al.* (2016), Olanrewaju and Oreyeni (2016) and Animasaun (2015).

In all the aforementioned studies, fluid properties were assumed to be constant throughout the boundary layer. It is a true fact that, the physical properties of the fluid may change appreciably with temperature. For instance, increase in the temperature as a result of the heat generated by the internal friction in heavy oils or engine oils of a moving vehicle affects the viscosity of the oil, therefore, the fluid viscosity can no longer be assumed constant. In addition to this fact, a slight increase in the temperature enhances the transport phenomena by reducing the viscosity across the momentum boundary layer, hence the heat transfer rate at the wall is also affected. It is believed that fluid properties that are more responsive to rise in temperature is viscosity and thermal conductivity. Sharma and Singh (2009) presented effects of variable thermal conductivity and heat source/sink on mhd flow near a stagnation point on a linearly stretching sheet. Uwanta*et al.* (2014) investigated effect of Jeffery fluid on heat and mass transfer past a vertical porous plate with Soret and variable thermal conductivity. Omowaye and Animasaun (2015) presented upper-convected Maxwell fluid flow with variable thermo-physical properties over a melting surface situated in hot environment subject to thermal stratification. Uwanta and Omokhuale (2014) studied effects of variable thermal conductivity on heat and mass transfer with Jeffery fluid.

The aim of this study is to unravel the influence of pertinent parameters on velocity, temperature, concentration of second grade fluid with variable viscosity, thermal conductivity and diffusivity within the boundary layer over a vertical surface with thermophoresis, chemical reactions and convective boundary condition. The governing partial differential equations are modified and converted to nonlinear ordinary differential equations using suitable similarity transformations. The resulting ODEs are solved by optimal homotopy analysis method. The effects of embedded flow controlling parameters on fluid velocity, temperature, concentration and

shear stress were presented graphically and analysed.

2.0 Mathematical Formulation

The introduction of Second grade fluid is to illustrate certain nonlinear effects that cannot be explained by the classical theory of Navier-Stokes for Newtonian fluids (see Truesdel and Noll (1965), Dunn and Fosdick (1974)). The constitutive law of incompressible homogeneous fluids of degree 2 is given by

$$\sigma = -pI + 2\vartheta A_1 + \alpha_1 A_2 + \alpha_2 A_1^2 \quad (1)$$

Where σ is the Cauchy tensor, p is the pressure, ϑ isthe viscosity α_1, α_2 are the normal stress modulli, A_1 and A_2 are the first two Rivlin-Ericksen tensors defined as

$$A_1(u) = \frac{1}{2}[\nabla u + \nabla u^T],$$

$$A_2 = \frac{DA_1}{D_t} + (\nabla u)^T A_1 + A_1 \nabla(\nabla u) \quad (2)$$

and $\frac{D}{Dt} = \partial_t + u.\nabla$ (3)

is the material derivative.

Dunn and Fosdick (1974) established that

$$\alpha_1 + \alpha_2 = 0, \ \alpha_1 \geq 0 \quad (4)$$

Writing the equation

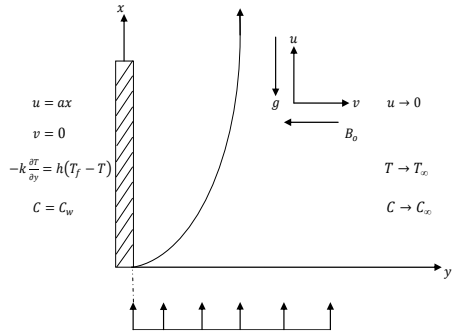


Fig. 1. Physical Configuration

For an incompressible fluid obeying the second grade fluid model with temperature dependent dynamic viscosity and thermal conductivity together concentration dependent diffusivity, the continuity, momentum, energy and concentration equations can be simplified by adopting the usual boundary layer theory approximations and then obtain

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (6)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu(T)\frac{\partial u}{\partial y}\right) - \frac{\alpha_1}{\rho}\left(\frac{\partial}{\partial x}\left(u\frac{\partial^2 u}{\partial y^2}\right) + \frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial y^2} + v\frac{\partial^3 u}{\partial y^3}\right) - \frac{\mu(T)}{\rho}\frac{1}{k}u - \frac{\sigma {B_o}^2 u}{\rho} + g[\beta_T(T - T_\infty) + \beta_C(C - C_\infty)] \quad (7)$$

Here the temperature dependent viscosity dynamical (μ) of the fluid can be expressed in the form Sivagnana *et al.* (2009)

$$\mu(T) = \mu^*\left[1 + \tau\left(T_f - T\right)\right] \quad (8)$$

The basic boundary layer equation for the system of heat transfer in the presence of temperature dependent thermal conductivity exponential and space dependent heat source is expressed as

$$u\frac{\partial T}{\partial x}+\ v\frac{\partial T}{\partial y}=\frac{1}{\rho C_p}\frac{\partial}{\partial y}\left(\kappa(T)\frac{\partial T}{\partial y}\right)+\frac{\kappa a}{\rho C_p \vartheta}\left[A\left(T_f-T_\infty\right)e^{-y\left(\frac{a}{\vartheta}\right)^{1/2}}+B(T-T_\infty)\right] \quad (9)$$

The mathematical model of temperature dependent thermal conductivity model of Charraudeau (1975), Salem and Fathy (2012) as

$$\kappa(T)=\kappa^*[1+\delta(T-T_\infty)] \quad (10)$$

The boundary layer equation in conformity to the mass transfer in the occurrence of thermophoresis and chemical reaction is obtained in form [7]

$$u\frac{\partial C}{\partial x}+\ v\frac{\partial C}{\partial y}=\frac{\partial}{\partial y}\left(D(C)\frac{\partial C}{\partial y}\right)-\frac{\partial}{\partial y}(V_T C)-K(C-C_\infty)^n \quad (11)$$

It will be valid to consider the model of variable concentration diffusivity given by the linear relation Hamad*et al.* (2011), El-Hawary *et al.* (2013)

$$D(C)=D^*[1+c(C-C_\infty)]=D^*(1+b\varphi) \quad (12)$$

The thermophoretic velocity V_T in equation (11) can be written in the form Talbot *et al. (1980)* as;

$$V_T=\ -\frac{\kappa\vartheta\nabla T}{T_{ref}}=\ -\kappa\vartheta\frac{1}{T_{ref}}\frac{\partial T}{\partial y}, \quad (13)$$

where $\kappa\vartheta$ represents the thermophoretic diffusivity, κ is the thermophoretic coefficient which ranges in value from 0.2 to 1.2 as indicated by Batchelor and Shen (1985) and is defined from the theory of Talbot *et al.*(1980)which is given by

$$\kappa=\frac{2C_s\left(\frac{\lambda_p}{\lambda_p}+C_tKn\right)\left[1+Kn\left(C_1+C_2exp^{-\frac{C_3}{Kn}}\right)\right]}{(1+3C_mKn)\left(1+\frac{2\lambda_g}{\lambda_p}+2C_tKn\right)} \quad (14)$$

Here, $C_1, C_2, C_3, C_m, C_s\ and\ C_t$ are constants, λ and λ_p are the thermal conductivities of the fluid and diffused particles respectively and Kn is the Knudsen number. A thermophoretic parameter τ can be defined (see Mills $et\ al.$ (1984) and Tsai (1999)) as;

$$\tau=\frac{\kappa(T_w-T_\infty)}{T_{ref}} \quad (15)$$

Equations (6), (7), (9)and (11) are subject to the following boundary conditions

$$u\ =u_w(x)=ax\,,\qquad v=0,\qquad -k\frac{\partial T}{\partial y}=h\left(T_f-T\right),\quad C=C_w\qquad at\ y=\ 0 \quad (16)$$

$$u\ \to U(x)=0\,,\qquad T\ \to T_\infty\,,\qquad C\ \to C_\infty\qquad as\qquad y\to\infty \quad (17)$$

where u and v are the velocity in x and y directions respectively, μ^* is the constant value of the coefficient, κ^* is the coefficient thermal conductivity, D^* is the coefficient concentration diffusivity, κ is the thermal conductivity, ρ is the density of the fluid second grade fluid. ϑ isthe kinematic viscosity,$\mu\ =\vartheta\rho$ is the dynamic viscosity,T is the fluid temperature in the boundary layer, C is the concentration of the species diffusion, D is the diffusion coefficient of the diffusing species, $\alpha=\frac{\kappa}{\rho C_p}$ is the thermal diffusivity.

The continuity equation (1) is satisfied by introducing a stream function ψ such that

$$u = \frac{\partial \psi}{\partial y} \quad , \qquad v = -\frac{\partial \psi}{\partial x} \tag{18}$$

The equations $(7), (9), (11), (16)$ and (17) becomes

$$\frac{\partial \psi}{\partial y}\frac{\partial}{\partial x}\frac{\partial \psi}{\partial y} - \frac{\partial \psi}{\partial x}\frac{\partial}{\partial y}\frac{\partial \psi}{\partial y} = \frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu(T)\frac{\partial}{\partial y}\left(\frac{\partial \psi}{\partial y}\right)\right) - \frac{\alpha_1}{\rho}\left(\begin{array}{c}\frac{\partial}{\partial x}\left(\frac{\partial \psi}{\partial y}\frac{\partial^2}{\partial y^2}\frac{\partial \psi}{\partial y}\right) - \\ \frac{\partial}{\partial y}\frac{\partial \psi}{\partial y}\frac{\partial^2}{\partial y^2}\frac{\partial \psi}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial^3}{\partial y^3}\frac{\partial \psi}{\partial y}\end{array}\right)$$

$$- \frac{\sigma {B_o}^2}{\rho}\frac{\partial \psi}{\partial y} - \frac{\mu(T)}{\rho}\frac{1}{k}\frac{\partial \psi}{\partial y} + g\beta_T(T - T_\infty) + g\beta_C(C - C_\infty) \tag{19}$$

$$\frac{\partial \psi}{\partial y}\frac{\partial T}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial T}{\partial y} = \frac{1}{\rho C_p}\frac{\partial}{\partial y}\left(\kappa(T)\frac{\partial T}{\partial y}\right) + \frac{\kappa a}{\rho C_p \vartheta}\left[A(T_w - T_\infty)e^{-y\left(\frac{a}{\vartheta}\right)^{1/2}} + B(T - T_\infty)\right] \tag{20}$$

$$\frac{\partial \psi}{\partial y}\frac{\partial C}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial C}{\partial y} = \frac{\partial}{\partial y}\left(D(C)\frac{\partial C}{\partial y}\right) - \frac{\partial}{\partial y}(V_T C) - K(C - C_\infty)^n \tag{21}$$

Subject to

$$\frac{\partial \psi}{\partial y} = u_w(x) = ax, \frac{\partial \psi}{\partial y} = 0, \quad -k\frac{\partial T}{\partial y} = h\left(T_f - T\right), \quad C = C_w \quad at \quad y = 0 \tag{22}$$

$$\frac{\partial \psi}{\partial y} \to 0, \qquad T \to T_\infty, \qquad C \to C_\infty\,, \qquad as \quad y \to \infty \tag{23}$$

The momentum, energy and concentration equations can be transformed into corresponding ordinary differential equations by the following transformations

$$\eta = y\frac{a^{\frac{1}{2}}}{\vartheta^{\frac{1}{2}}}, \quad \frac{\psi(x,y)}{x a^{\frac{1}{2}}\vartheta^{\frac{1}{2}}} = f(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_f - T_\infty}, \quad \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty} \tag{24}$$

Thus, u and v are given by $u = ax\frac{df}{d\eta}$, $v = -a^{\frac{1}{2}}\vartheta^{\frac{1}{2}}f(\eta)$, substituting$(24)$ into equations$(19) - (23)$, we obtain

$$[1 + (1 - \theta)\xi\frac{d^3 f}{d\eta^3} + f(\eta)\frac{d^2 f}{d\eta^2} - \left(\frac{df}{d\eta}\right)^2 - \xi\frac{d^2 f}{d\eta^2}\frac{d\theta}{d\eta} - K\left(2\frac{df}{d\eta}\frac{d^3 f}{d\eta^3} - \left(\frac{d^2 f}{d\eta^2}\right)^2 - f\frac{d^4 f}{d\eta^4}\right)$$

$$-P_s[1 + (1 - \theta)\xi]\frac{df}{d\eta} - M\frac{df}{d\eta} + \; G_r\theta + G_c\phi = 0 \tag{25}$$

$$[1 + \theta\varepsilon]\frac{\mathrm{d}^2\theta}{\mathrm{d}\eta^2} - \mathrm{P_r}\theta\frac{\mathrm{df}}{\mathrm{d}\eta} + \mathrm{P_r f}(\eta)\frac{\mathrm{d}\theta}{\mathrm{d}\eta} + \varepsilon\frac{\mathrm{d}\theta}{\mathrm{d}\eta}\frac{\mathrm{d}\theta}{\mathrm{d}\eta} + \left[\mathrm{A}e^{-y\left(\frac{a}{\vartheta}\right)^{1/2}} + \mathrm{B}\theta\right] = 0 \tag{26}$$

$$[1 + \phi\omega]\frac{d^2\phi}{d\eta^2} + \omega\left(\frac{d\phi}{d\eta}\right)^2 + S_C f(\eta)\frac{d\phi}{d\eta} - S_c\tau\left(\frac{d\theta}{d\eta}\frac{d\phi}{d\eta} + \phi(\eta)\frac{d^2\theta}{d\eta^2}\right) - S_c\lambda\phi^n = 0 \tag{27}$$

The corresponding boundary conditions take the form

$$f'(0) = 1, \quad f(0) = 0, \quad \frac{d\theta}{d\eta} = -\gamma\left(1 - \theta(0)\right), \quad \phi(0) = 1 \quad at \quad \eta = 0 \tag{28}$$

$$f'(\eta) \to 0\,, \quad \theta(\eta) \to 0, \quad \phi(\eta) \to 0 \quad as \quad \eta \to \infty \tag{29}$$

In the equations above η is the independent dimensionless similarity variable, A and B are the heat source parameters, $K = \frac{\alpha_1 a}{\rho\vartheta}$ is the second grade parameter, $M = \frac{\sigma B_o^2}{\rho a}$ is the magnetic field parameter, $P_r = \frac{\vartheta}{\alpha}$ is the

Prandtl number, $P_s = \frac{\vartheta}{ka}$ is the porosity parameter, $G_r = \frac{g\beta_T(T_w - T_\infty)}{a^2 x}$ is modified thermal Grashof number, $G_c = \frac{g\beta_C(C_w - C_\infty)}{a^2 x}$ is modified solutal Grashof number, $\xi = b(T_f - T_\infty)$ is the temperature-dependent viscous parameter, $\varepsilon = \delta(T_f - T_\infty)$ is the temperature dependent variable thermal conductivity parameter, $w = q(C_w - C_\infty)$ is the variable concentration diffusivity, $\lambda = \frac{k_n(C_w - C_\infty)^{n-1}}{a}$ is the dimensionless chemical reaction parameter, $S_c = \frac{\vartheta}{D}$ is the Schimdt number.

The physical quantities of interest are the skin friction coefficient C_f, Local Nusselt number Nu_x and Local Sherwood number $Sh_x$are defined as

$$C_f = \frac{\tau_w}{\rho(u_w)^2}, \quad Nu_x = \frac{xq_w}{\kappa(T_f - T_\infty)}, \quad Sh_x = \frac{xq_m}{\kappa(C_\infty - T_\infty)} \tag{30}$$

$$\tau_w = \mu\left[\frac{\partial u}{\partial y}\right]_{y=0} = \left[\mu\frac{\partial u}{\partial y} + \alpha_1\left(2\frac{\partial u}{\partial x}\frac{\partial u}{\partial y} + u\frac{\partial^2 u}{\partial x \partial y}\right)\right]_{y=0} \tag{31}$$

whereτ_w is thewall shear stress, q_w and q_m denote the surface heat flux and the mass flux from the surface.

3.0 Optimal Homotopy Analysis Solution

Invoking the rule of solution expressions above for $f_o(\eta), \theta_o(\eta)$ and $\phi_o(\eta)$ on $(25) - (27)$ together with boundary conditions (28)and(29), the initial guesses$f_o(\eta)$, $\theta_o(\eta)$ and $\phi_o(\eta)$ which satisfies both the initial and the boundary conditions (28) and (29)

$$f_o(\eta) = 1 - \exp(-\eta), \qquad \theta_o(\eta) = \frac{\gamma e^{-\eta}}{1+\gamma}, \qquad \phi_o(\eta) = e^{-\eta} \tag{32}$$

Linear operators L_f , L_p and L_θ are:

$$L_f[f(\eta, q)] = \frac{\partial^3 f(\eta; q)}{\partial \eta^3} - \frac{\partial f(\eta; q)}{\partial \eta} \tag{32}$$

$$L_\theta[\theta(\eta, q)] = \frac{\partial^2 \theta(\eta; q)}{\partial \eta^2} - \theta(\eta; q) \tag{33}$$

$$L_\phi[\phi(\eta, q)] = \frac{\partial^2 \phi(\eta; q)}{\partial \eta^2} - \phi(\eta; q) \tag{34}$$

The operators L_f , L_p and $L_\theta$have the following properties

$$L_f[C_1 + C_2\exp(-\eta) + C_3\exp(\eta)] = 0, \qquad L_p[C_4\exp(-\eta) + C_5\exp(\eta)] = 0\,,$$
$$L_\theta[C_6\exp(-\eta) + C_7\exp(\eta)] = 0 \tag{35}$$

In which C_1 , C_2, C_3 , C_4 , C_5 , C_6 and C_7 are arbitrary constants.

If $q \in [0,1]$ denotes an embedding parameter, $\hbar_f, \hbar_\theta$and $\hbar_\phi$ the non-zero auxiliary parameters then, the zeroth order of deformation problems are constructed as

$$(1-q)L_f[f(\eta; q) - f_o(\eta)] = q\hbar_f H_f(\eta) N[f(\eta; q), p(\eta; q), \theta(\eta; q)] \tag{36}$$

$$(1-q)L_\theta[\theta(\eta; q) - \theta_o(\eta)] = q\hbar_\theta H_\theta(\eta) N[f(\eta; q), p(\eta; q), \theta(\eta; q)] \tag{37}$$

$$(1-q)L_\phi[\phi(\eta; q) - \phi_o(\eta)] = q\hbar_\phi H_\phi(\eta) N[f(\eta; q), p(\eta; q), \phi(\eta; q)] \tag{38}$$

Subject to boundary conditions

$$f(\eta = 0; q) = 0\,, \frac{\partial f(\eta = 0; q)}{\partial \eta} = 1, \theta(\eta = 0; q) = -\gamma[1 - \theta(0; q)], \phi(\eta = 0; q) = 0 \tag{38}$$

$$\frac{\partial f(\eta \to \infty; q)}{\partial \eta} \to 0\,, \quad \theta(\eta \to \infty; q) \to 0, \quad \phi(\eta \to \infty; q) \to 0 \tag{39}$$

where the nonlinear operators are defined as

$$N_f[f(\eta; q), \theta(\eta; q), \phi(\eta; q)] = [1 + \big(1 - \theta(\eta; q)\big)\xi\frac{\partial^3 f(\eta; q)}{\partial \eta^3} - f(\eta; q)\frac{\partial^2 f(\eta; q)}{\partial \eta^2} -$$

$$\frac{\partial^2 f(\eta;q)}{\partial \eta^2} - \xi \frac{\partial^2 f(\eta;q)}{\partial \eta^2}\frac{\partial \theta(\eta;q)}{\partial \eta} - K\left\{\begin{matrix} 2\dfrac{\partial f(\eta;q)}{\partial \eta}\dfrac{\partial^3 f(\eta;q)}{\partial \eta^3} - \left(\dfrac{\partial^2 f(\eta;q)}{\partial \eta^2}\right)^2 \\ -f(\eta;q)\dfrac{\partial^4 f(\eta;q)}{\partial \eta^4} \end{matrix}\right\} -$$

$$M\frac{\partial f(\eta;q)}{\partial \eta} + G_r\theta(\eta;q) + G_c\phi(\eta;q) = 0 \quad (40)$$

$$N_\theta[f(\eta;q),\theta(\eta;q),\phi(\eta;q)] = [1+\theta(\eta;q)]\frac{\partial^2 f(\eta;q)}{\partial \eta^2} - P_r S_t \frac{\partial f(\eta;q)}{\partial \eta} -$$

$$P_r\theta(\eta;q)\frac{\partial f(\eta;q)}{\partial \eta} + P_r f(\eta;q)\frac{\partial \theta(\eta;q)}{\partial \eta} + \varepsilon\frac{\partial \theta(\eta;q)}{\partial \eta}\frac{\partial \theta(\eta;q)}{\partial \eta} +$$

$$[\mathrm{Ae}^{-\mathrm{y}\left(\frac{a}{\vartheta}\right)^{\frac{1}{2}}} + \mathrm{B}\theta(\eta;\mathrm{q})] \quad (41)$$

$$N_\phi[f(\eta;q),\theta(\eta;q),\phi(\eta;q)] = [1+\omega\phi(\eta;q)]\frac{\partial^2 \phi(\eta;q)}{\partial \eta^2} + \omega\frac{\partial \phi(\eta;q)}{\partial \eta}\frac{\partial \phi(\eta;q)}{\partial \eta} +$$

$$S_c f(\eta;q)\frac{\partial \phi(\eta;q)}{\partial \eta} - S_c\tau\left(\frac{\partial \theta(\eta;q)}{\partial \eta}\frac{\partial \phi(\eta;q)}{\partial \eta} + \phi(\eta;q)\frac{\partial^2 \phi(\eta;q)}{\partial \eta^2}\right) - S_c\lambda\phi(\eta;q)^n \quad (42)$$

where q is an embedding parameter, $\hbar_f, \hbar_\theta$ and $\hbar_\phi$ are the non-auxiliary parameters and N_f, N_θ and N_ϕ are the non-linear operators. $q = 0$ and $q = 1$ we have;

$$f(\eta;0) = f_o(\eta), \quad \theta(\eta;0) = \theta_o(\eta), \quad \phi(\eta;0) = \phi_o(\eta) \quad \text{and}$$
$$f(\eta;1) = f_o(\eta), \quad \theta(\eta;1) = \theta_o(\eta), \quad \phi(\eta;1) = \phi_o(\eta) \quad (43)$$

and $f(\eta;q), \theta(\eta;q)$ and $\phi(\eta;q)$ vary from $f_o(\eta), \theta_o(\eta)$ and $\phi_o(\eta)$ when q varies from 0to 1.
By Taylors series expansion one has;

$$f(\eta;q) = f_o(\eta) + \sum_{m=1}^{\infty} f_m(\eta)q^m \quad (44)$$

$$\theta(\eta;q) = \theta_o(\eta) + \sum_{m=1}^{\infty}\theta_m(\eta)q^m \quad (45)$$

$$\phi(\eta;q) = \phi_o(\eta) + \sum_{m=1}^{\infty}\phi_m(\eta)q^m \quad (46)$$

Clearly, the convergence of series equations $(36)-(38)$ is closely associated with $\hbar_f, \hbar_\theta$ and $\hbar_\phi$. The auxiliary parameter $\hbar_f, \hbar_\theta$ and $\hbar_\phi$ are chosen such that the series equations $(36)-(38)$ converge at$q = 1$. Hence,

$$f(\eta;q) = f_o(\eta) + \sum_{m=1}^{\infty} f_m(\eta)q^m \quad (47)$$

$$\theta(\eta;q) = \theta_o(\eta) + \sum_{m=1}^{\infty}\theta_m(\eta)q^m \quad (48)$$

$$\phi(\eta;q) = \phi_o(\eta) + \sum_{m=1}^{\infty}\phi_m(\eta)q^m \quad (49)$$

If we denote the special solution f_m^*, θ_m^* and ϕ_m^* then the general solutions f_m, θ_m and ϕ_m are

$$f_m(\eta) = f_m^*(\eta) + C_1 + C_2\exp(\eta) + C_3\exp(-\eta) \quad (50)$$

$$\theta_m(\eta) = \theta_m^*(\eta) + C_4 + C_5\exp(\eta) + C_6\exp(-\eta) \quad (51)$$

$$\phi_m(\eta) = \phi_m^*(\eta) + C_7\exp(\eta) + C_8\exp(-\eta) \quad (52)$$

Here, $f_m^*(\eta), \theta_m^*(\eta)$and $\phi_m^*(\eta)$ are the particular solutions of equations $(47)-(49)$. Following the rule of expression, the rule of coefficient ergodicity and the rule of solution existence as discussed in Liao (2003) we choose auxiliary functions as

$$H_f = H_\theta = H_\phi = 1 \quad (53)$$

4.0 Convergence of the Optimal Homotopy Solutions

It is obvious that the series $(44)-(46)$ consist of the non-zero auxiliary parameters $\hbar_f, \hbar_\theta$ and $\hbar_\phi$ which can adjust and control the

convergence. The interval on $\hbar$-curve becomes parallel to the $\hbar$-axis is recognized as the set of admissible values of $\hbar_f, \hbar_\theta$ and $\hbar_\phi$ for which the solutions series converges. These figures show that the range for the acceptable values of $\hbar_f, \hbar_\theta$ and $\hbar_\phi$ are $-0.75 \leq \hbar_f - 0.35$, $-3.50 \leq \hbar_\theta - 1.50$ and $-0.80 \leq \hbar_f - 0.40$. Obviously, from the $\hbar$-curves for this problem, we obtained the approximate optimal values of $\hbar_f, \hbar_\theta$ and $\hbar_\phi$ at $10th$-order of approximations as $-1.2564, -0.3507$ and -1.0344

5.0 Results and Discussion

To analyse our results, computation has been conducted using the approximate analytic method described above for various values of second grade parameter$K$, temperature dependent viscosity parameterξ, temperature dependent thermal conductivity ε, concentration dependent diffusivity parameter ω, Biot number γ, thermophoretic parameterτ, modified buoyancy parameters G_r, G_c, and chemical reaction parameter λ.Table 1 provides numerical value of skin friction coefficients for the values of K, P_r, ξ and ε. It is observed that for the first two entries of ξ there is a slight effect on skin friction coefficients and for the last four entries of Second grade parameter K while other parameters were fixed at their respective values,there is a noticeable increase in the skin friction coefficient.The numerical computation of reduced Nusselt numbers $Nu_x Re_x^{-1/2}$ for the values of K, P_r, ξ and ε are presented in Table 2.For the first three entries of Prandlt number $P_{r,}$ there is a significant decreasein the reduced Nusselt numbers, while increase in $\xi$from 0.4 to 0.8 increases the reduced Nusselt numbersand also for the last three entries of K whenP_r, ξ and ε were fixed, there is an obvious increase in the reduced Nusselt numbers. Table 3 shows the numerical value of reduced Sherwood numbers $Sh_x Re_x^{-1/2}$ for values of K, ξ, ε and τ. From the table, it is observed that for fixed values of K, ξ andε, increase in thermophoretic parameterτ corresponds to decrease in the reduced Sherwood numbers. Now, we shall direct our attention to the graphical results discussion that provides more understanding about the problem under analysis. Dual solutions have been obtained for both cases of assisting flow $G_r = G_c > 0$ and opposing flow $G_r = G_c < 0$. In order to illustrate the results graphically, the numerical values are plotted in Figs. $2 - 16$. The dual solutions of the effects of the second grade parameter K are presented in Figs. $(2) - (5)$.

Table 1. Numerical values of skin friction coefficients for various values of K, P_r, ξ, and ε

K	P_r	ξ	ε	$-f''(0)$
0.1	1.0	0.4	0.3	0.5550
0.1	1.0	0.7	0.3	0.5753
0.3	5.0	0.7	0.3	0.6499
0.7	5.0	0.7	0.3	1.0761
1.0	5.0	0.7	0.3	1.4839
1.2	5.0	0.7	0.3	1.9103

It is observed from Fig. (2) that as K increases from $0, 0.3$ through 0.7 to 1.0for the case of assisting flow$G_r = G_c > 0$, the velocity of the second grade fluid decreases within the domain $0 \leq \eta \leq 3.64$. At the specific region of $\eta = 3.68$ all curves merge and decay far away from the heated vertical wall. It is also observed from Fig. (3) that increasing K corresponds to decrease in the temperature of the fluid for the case of assisting flow.

Table 2. Numerical values of reduced Nusselt numbers for various values of K, P_r, ξ, and ε

K	P_r	ξ	ε	$-\theta'(0)$
0.1	0.1	0.2	0.4	0.0834
0.1	0.5	0.2	0.4	0.0590
0.1	0.7	0.2	0.4	0.0175
0.1	0.7	0.4	0.4	0.0175
0.1	0.7	0.6	0.4	0.0221
0.1	0.7	0.8	0.4	0.0230
0.3	0.7	0.8	0.4	0.0247
0.6	0.7	0.8	0.4	0.0303
0.9	0.7	0.8	0.4	0.0591

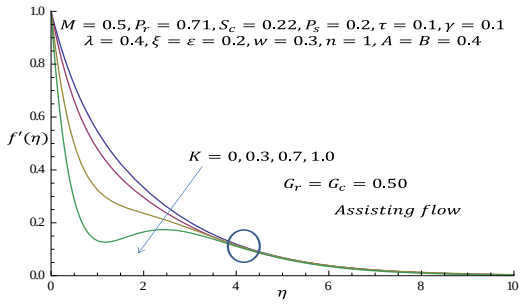
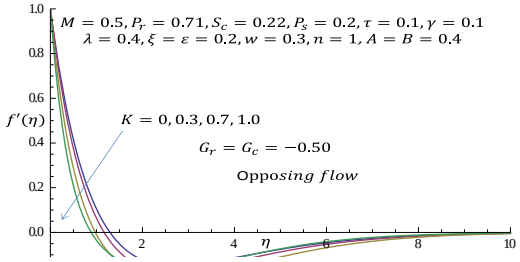


Fig. 2. Effect of K on velocity profile when **profile**

$$G_r = G_c = 0.5$$

profileType equation here.



$$G_r = G_c = -0.5$$

Fig. 4. Effect of K on velocity profile when

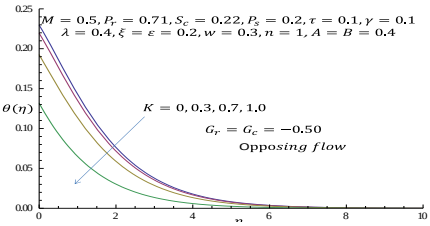


Fig. 5. Effect of K on temperature profile when **profile**

$$G_r = G_c = -0.5$$

Likewise, it is observed from Fig. (4)for the case of opposing flow $G_r = G_c < 0$, that increase in the values of K results to decrease in the velocity profile near the wall to a point far from the wall. It can also be observed that as K increases from 0 to 1.0, the velocity profile over shoot downward towards the negative values, and this suggests that as$G_r, G_c < 0$, there is generation of heat energy that cannot subjugate equivalence influence of the cooling of the fluid at the surface, hence there is a mammoth increase in the viscosity of the second grade fluid as it flows in the boundary layer. As a result of this effect, the temperature of the fluid decreases as K is increased as shown in Fig. (5). The distribution of velocity for various values of temperature dependent viscous parameter ξ for the case of $G_r = G_c > 0$ when the values of other flow parameters are kept constant is described in Fig. (6). It is clearly observed that the magnitude of the vertical velocity component increases significantly as

ξincreases, and the velocity boundary layer is developed as the fluid flow over the surface when modified buoyancy parameters $G_r, G_c > 0$.Generally, what this observation connotes in addition to obtain an understanding of the physical mechanisms that motivates natural convection transfer is that in free convection flows bounded by a surface with boundary layer development on a heated vertical plate, the fluid far from the wall is quite more denser than the fluid close to the wall, making it possible for the fluid close to the wall which appears lighter to flow at an increased velocity upward towards the quiescent region. Owing to this fact, buoyancy forces stimulate a natural convection boundary layer in which the heated fluid rises vertically and it is able to surmount hindering impact of viscosity. The effect of increasing ξon wall shear stress is elucidated in Fig. (7) It is seen from the figure that as ξ increases, the wall shear stress also increases. Fig. (8) depicts the influence of ξ on velocity distribution when the modified buoyancy parameters $G_r, G_c < 0$.The detailed examination shows that as $\xi = b(T_f - T_\infty)$ increases from $0, 2.0$ through 4.0 to 6.0, with the modified buoyancy parameters taking negative value i.e. $G_r, G_c < 0$, the velocity profile increases but when $\xi = 0$ and 2.0 there is an obvious effect on the velocity distribution tends to attain a negative value within the domain $2.5 \leq \eta \leq 8.5$ and $1.5 \leq \eta \leq 8.5$, but as $\xi = b(T_f - T_\infty)$ increases from 4.0 to 6.0 there is sufficient heat energy required to breakdown the strong intermolecular bonds in the fluid boundary and the retarding influence of the viscosity is subdued making the velocity of the second grade fluid to increase and hence, the velocity profile take a positive value at $\xi = b(T_f - T_\infty) =$ 4.0 and 6.0.

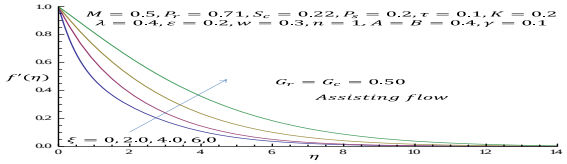


Fig. 6. Effect of ξ on velocity profile when $G_r = G_c = 0.5$

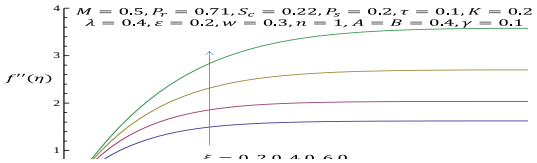
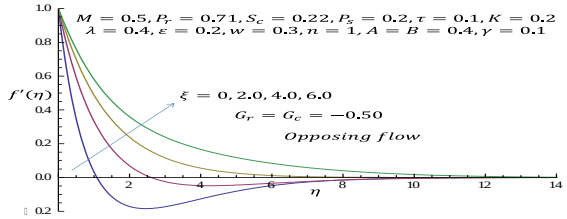
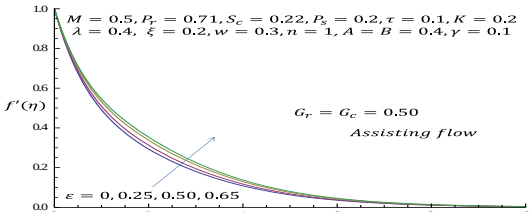


Fig. 7. Effect of ξ on shear stress profile when $G_r = G_c = 0.5$

Fig. (9) presents the distribution of the velocit conductivity parameter ε when modified buoyancy



$G_r = G_c = -0.5$



$G_r = G_c = 0.5$

 https://doi.org/10.57233/ijsgs.v4i2.179

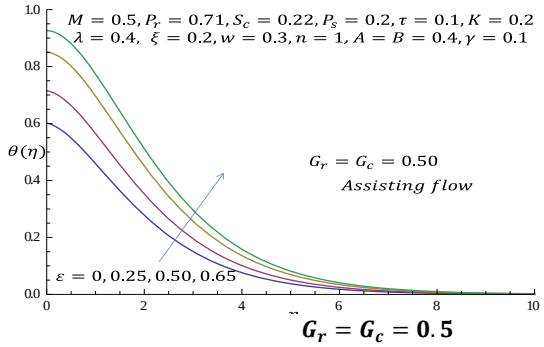


Fig. 10. Effect of ε on temperature profile when

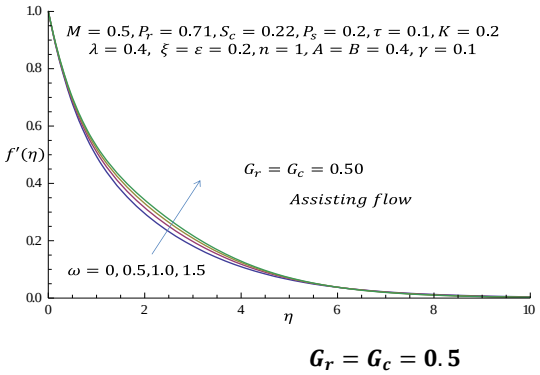


Fig. 11. Effect of ω on velocity profile when

It is observed that increase in ε corresponds to increase in the velocity profile. Physically, the heated vertical plate transfer heat energy to the fluid layer very close to the a surface and hence, the fluid rises and the temperature of the second grade fluid increases as well as shown in Fig.(10). The effect of variable concentration diffusivity parameter ω when $G_r, G_c > 0$on velocity profile is described in Fig. (11). It is observed that as ω increases there is a slight increase in the velocity profile. Likewise, from Fig. (12), increase in ω leads to significant increase in the concentration profile. Fig. (13) illustrates the effect of Biot number γ on the temperature profile. It seen that the temperature reduces as the γ increases. Also, the peak temperature occurs in the thermal boundary layer in a region close to the plate.The influence of thermophoresis parameter τ on concentration distribution when $\lambda = 0.6$ at first orderchemical reaction i.e. $n = 1$ and $G_r, G_c > 0$ is illustrated in Fig. (14).

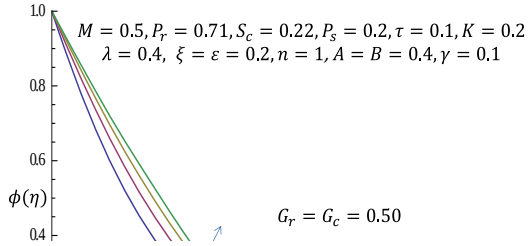


Fig. 12. Effect of ω on concentration profile when

$G_r = G_c = 0.5$

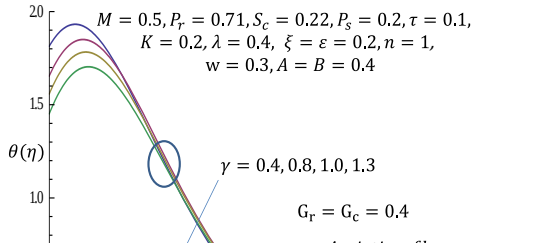


Fig. 13. Effect of γ on temperature profile when

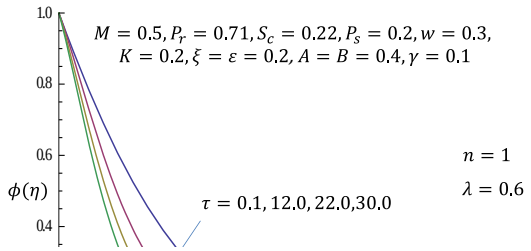


Fig. 14. Effect of τ on concentration profile when

$n = 1$

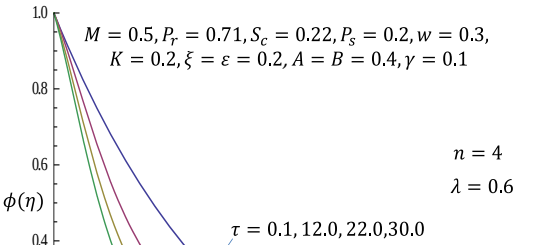


Fig. 15. Effect of τ on concentration profile when

$n = 4$

the wall where there is decreasing temperature gradient (i.e. they move to a region where there is low heat energy). Likewise, the same effect is observed from Fig (15) where the case of 4^{th} order chemical reaction is considered. It is clear enough that the concentration profile decreases with incrrease in τ even for higher order of chemical reaction. Hence, the effect of of increasing the thermophoretic parameter τ is limited to decreasing the distribution of the concentration.

Table 3. Numerical values of Sherwood numbers for various values of K, ξ, ε and τ

K	ξ	ε	τ	$-\phi'(0)$
0.2	0.3	0.5	0.1	0.3334
0.2	0.3	0.5	0.3	0.3317
0.2	0.3	0.5	0.5	0.3301
0.2	0.3	0.5	0.7	0.3284

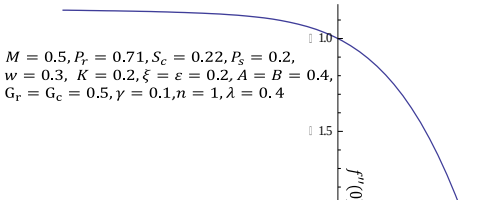


Fig. 15. $\hbar$-curve of $f''(0)$ obtained at $10th$-order of approximation

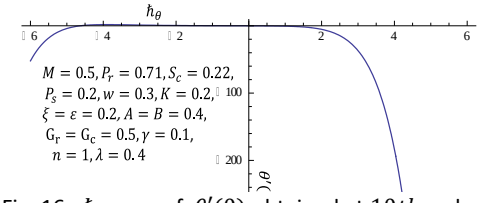
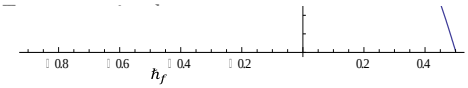


Fig. 16. approximation



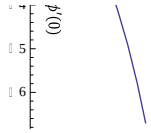


Fig. 17. $\hbar$-curve of $\phi'(0)$ obtained at $10th$-order of approximation

6.0 Conclusion

We have analytically investigated steady, incompressible second grade fluid flow past a vertical surface with convective boundary condition. The corresponding influences of thermophoresis, variation in viscosity, thermal conductivity due to temperature and variation in diffusivity due to concentration are properly considered. The governing partial differential equations are transformed into nonlinear ordinary differential equations by using similarity transformation. Analytic approach was utilized to study the behavior of all the controlling parameters on the flow velocity and temperature profiles in the boundary layer. The main points of the present study can be summed up as follows;

1. Velocity and temperature profiles are decreasing functions of second grade parameter K for both cases of modified buoyancy parameters $G_r, G_c > 0$ and $G_r, G_c < 0$.
2. Within the boundary layer of the problem in consideration, velocity profile is an increasing function of temperature dependent viscous parameter ξ for both cases of $G_r, G_c > 0$ and also$G_r, G_c < 0$, but velocity takes negative value for the case of $G_r, G_c < 0$ as temperature

dependent viscous parameter ξ increases.

3. The wall shear stress increases with increasing temperature dependent viscous parameter ξ when $G_r, G_c > 0$.
4. Both the velocity and temperature profiles are increasing functions of temperature dependent thermal conductivity parameter ε when $G_r, G_c > 0$.
5. The velocity and concentration profiles increase with increase in concentration dependent diffusivity parameter ω
6. Temperature of second grade fluid increases with increase in Biot numberγ.
7. Concentration distribution is a decreasing function of thermophoretic parameter τ for different orders of chemical reactions i.e. $n = 1, 4$.

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