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Application of Queuing Theory in a University Clinic

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ABSTRACT

Institutions such as banks, airlines, hospitals and telecommunication industries employ queuing theory to help in assisting the capacity levels needed to experience demand in a more optimized way for service improvement. In this article, we studied and employed the queuing theory activities of the Federal University Gusau Health Services Clinic. The distributions of patients' arrival and service times were identified. The system was treated as a single, time-independent arrival with multiple service points. Based on the data obtained on the arrival and service processes, appropriate distributions were fitted and tested using the Chi-squared goodness of fit-test.

Keywords: Arrival time; Chi-squared goodness of fit test; Patients; Service time and Queuing

1.0 INTRODUCTION

Arrival events and service time are two different things that are probabilistic in nature. In other words, there is a measure of uncertainty in their individual distributions. This idea of uncertainty confirms the need for probability and statistics in analyzing queuing circumstances or procedures. A queuing process is given birth to when the arrival process and the service process are pooled.

The arrival process studies how events or individuals emanate to a point, usually a service facility. It is assumed to have a Poisson distribution. The service process, on the other hand, has to do with waiting in time while receiving a service. Service time is expected to have an exponential distribution or rather, a modified form of it, which may be the Gamma or Erlang distributions, depending on the service mechanism.

Queuing theory is the mathematical approach to the analysis of waiting lines in any setting where the arrival rate of subjects is faster than what the system can handle (Nkeiruka *et al.*, 2013). Nkeiruka *et al.* (2013) was a cross-sectional survey where questionnaires were administered to patients who attended the general outpatient department. Observations were also made on the queuing model and service discipline at the clinic. A total of 210 patients were surveyed. Most of the patients spent 2 hours or less in the queue before being seen by a doctor and less than 1 hour to see the doctor. The theory started with research by the Danish Engineer A.K. Erlang who is called the father of queuing theory and who published his articles on the study of congestion in telephone traffic (Shanmugasundaram and Umarani, 2015). Queuing theory is the branch of operation research that explores the relationship between the demand on a service system and the delays suffered by the users of the system (Ogunlade, 2019).

Oche and Adamu (2013) used a cross-sectional study among new patients attending the GOPD of the Usmanu Danfodio University Teaching Hospital ,Sokoto, Nigeria. In the research, which involved 100 patients, 61% of the respondents waited for 90-180 minutes in the clinic before seeing a doctor. The Institute of Medicine (IOM) recommends that, at least 90% of patients should be seen within 30 min of their scheduled appointment time. Malley (1983) This is, however, not the case in most developing countries, as several studies have shown that patients spend 2-4 hours in outpatient

departments before seeing the doctor. In the research of (Singh *et al.*, 1999, Ofilli and Ofowve, 2005 and Ofilli and Ofowve, 2007) sources of dissatisfaction with health care reported by patients include having to wait a long period of time in the clinic, Dansky and Miles (1997) and several studies have documented the negative association between increased waiting time and patient satisfaction with primary care. In the studies of (Huang, 1994 and Average, 2008) gave the duration of waiting time varies from country to country, and even within a country it varies from center to center.

Long waiting times have been reported in both developed and developing countries. In the USA, an average waiting time of about 60 min was found in Atlanta and an average of 188 min in Michigan. Dos Santos *et al.* (1994) revealed that in Nigeria, an average waiting time of about 173 min was found in Benin, Dansky and Miles (1997) while in University College Hospital Ibadan, a mean waiting, which is a time of 1 hour 13 minutes was observed. Bamboye *et al.* (1994) studied the time spent waiting is a resource investment by the patient for the desired goal of being seen by the physician and therefore may be moderated by the outcome. Adeniran *et al.* (2022) studied the service efficiency of the Ahmadu Bello University Teaching Hospital Zaria, Kaduna State, using multiple server queuing models. Over a two-week period, primary data was collected at the hospital using observation and questionnaire methods to determine the queuing model that minimises patient waiting time. The findings revealed that a significant proportion of patients were dissatisfied with the hospital's queue management strategies. Majlesinasab *et al.* (2022) proposed a framework that employs queuing theory and location analysis to address both activation and travel times. In this regard, first, queuing theory is used to evaluate an EMS call center's performance and reduce the average activation time. Three different mathematical models, mobile stations, MEXCLP, and MECRP, were used to locate the ambulances in order to minimise mean travel time.

Mtonga *et al.* (2022) considered an adaptive staff schedule at Outpatient Department of Ntaja Health Center in Malawi-A queuing theory application. We use queuing theory methods to analyse and evaluate the facility's relationship between staffing ratios and waiting times. A model of queuing variables and approximate suggested staffing ratios using patient flow data from a rural clinic in Malawi to achieve a steady state, leading to lower waiting times and, as a result, improved service delivery at the clinic. Abdellatif and Abdelatey (2022) used operations management technique, the queuing theory model was used to calculate the number of required beds that must be available to cover all recorded cases. Based on the information available, Italy was selected as a case study because it was one of the hotspot countries at the start of the crisis, following China. From the beginning of the crisis to more than a year later, an analysis of the behaviour of hospital waiting lines is presented. The number of required hospital and ICU beds is determined using queueing theory formulas.

Oladimeji and Ibidoja (2020) studied the distribution of service time of patients, the distributions of patient arrival and service time were also studied. The system was treated as a single, time-independent arrival with multiple service points. Based on the histograms obtained for the arrival and service processes, appropriate distributions were fitted and tested using the Chi-squared goodness of fit test. It was discovered that the arrival process follows a Poisson distribution with an arrival rate of 0.8 patients every 2 minutes, whereas the service process follows a gamma distribution with shape and scale parameters of 6 and 0.1, respectively. Suleiman *et al.* (2022) studied the queuing performance characteristics were calculated using the Microsoft Excel package. Data used in this study were obtained through observations and personal interviews using eight hospitals in Katsina, Kaduna, Sokoto, and Zamfara States in North-West Nigeria. The results show that the General Hospital Hunkuyi is the busiest, due to the highest utilisation factor as well as the most patients in the queue. However, Ahmadu Bello University Teaching Hospital in Zaria is the least busy, and the Federal Medical Centre in Katsina has the fewest patients in line. Aziati and Hamdan (2018) developed a suitable model which used descriptive analytical and simulation methods. The arrival rate and service rate of patients at the outpatient counter are used to collect waiting time for this study. Microsoft Excel was used to calculate and analyse the data. Using the ARENA software, the queuing system of the patient's current situation was modelled and simulated based on the analysed data. The simulation model results show that the average waiting time for patients in the queue is 54.295 minutes, with a total of 327 patients exiting. The average service time is thus 13.481 minutes.

2.0 MATERIALS AND METHODS

The primary data were collected from Federal University Gusau Health Services Clinic. Data on 537 patients were collected between June 25th 2019 and 23rd July 2019. The system was treated as a single, time-independent arrival with multiple service points. The arrival time, waiting time and service time were obtained by the observation method. The data obtained from this study were entered on Microsoft Excel and SPSS was employed for the analysis. The descriptive statistics were computed for waiting time and service time, histograms, Normal Q-Q plot were also plotted, this is to have deep understanding of the results.

The Chi Squared goodness of fit test is a particular useful technique for testing whether the observed sample distribution data are representative of a particular distribution. The actual frequencies are compared with the frequencies that theoretically would be expected to occur if the data followed the specific distribution of interest.

The test statistic is given as

$$X^2 = \frac{\sum_{i=1}^n (O_i - E_i)^2}{E_i} \sim \chi^2_{k-p-1} \text{ degrees of freedom.}$$

where

O_i = Observed frequency

E_i = Expected frequency

P = Number parameters estimated

K = Number of groups or categories

Decision Rule: Reject H₀ if $X^2 > \chi^2_{k-p-1, \alpha}$ or if P-value $< \alpha$ otherwise do not reject.

3.0 DESCRIPTIVE STATISTICS

Descriptive statistics help to describe and understand the features of specific data set by giving short summaries about the sample and measures of data. All descriptive statistics are either measures of central tendency or measures of variability.

Table 1: Descriptive Statistics

	N	Range	Minimum	Maximum	Mean	Std. Deviation	Variance
Waiting Time	537	43	2	45	11.54	6.809	46.361
Service Time	537	48	2	50	5.40	3.006	9.039

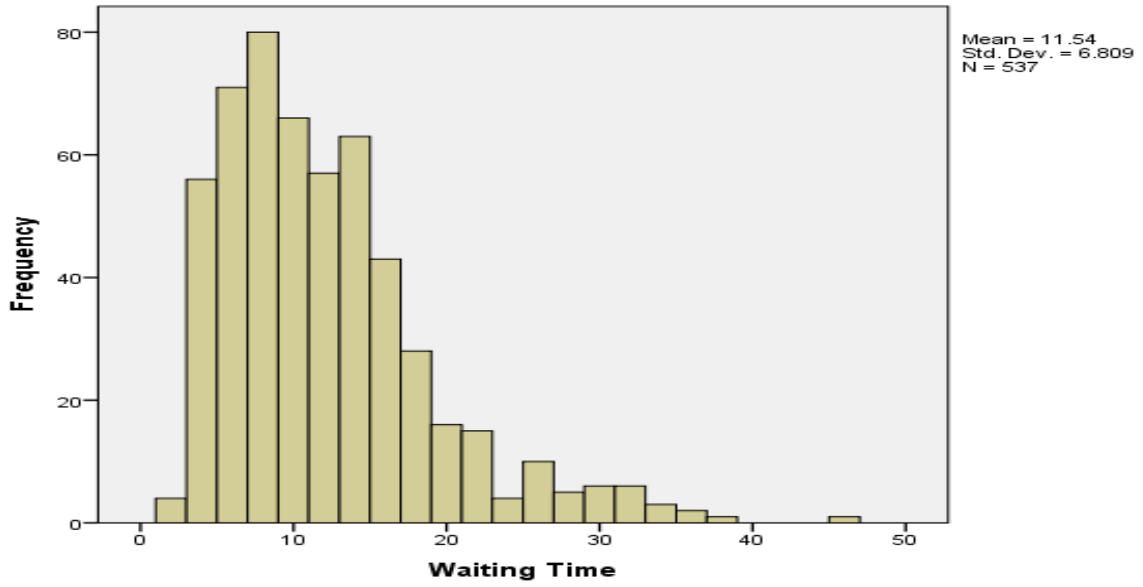


Figure 1: Histogram for Waiting Time

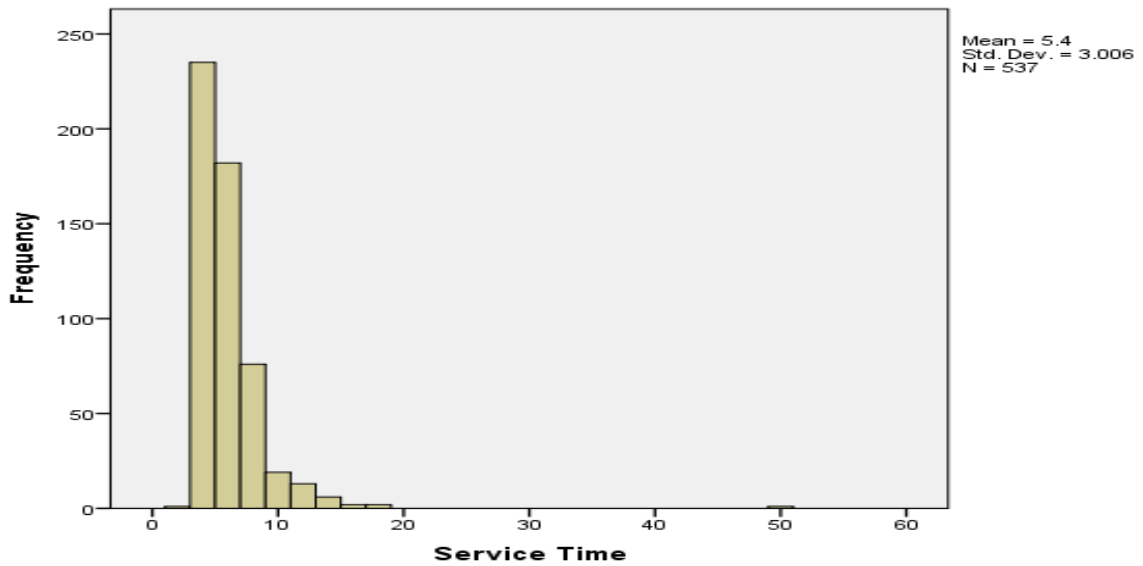


Figure 2: Histogram for Service Time

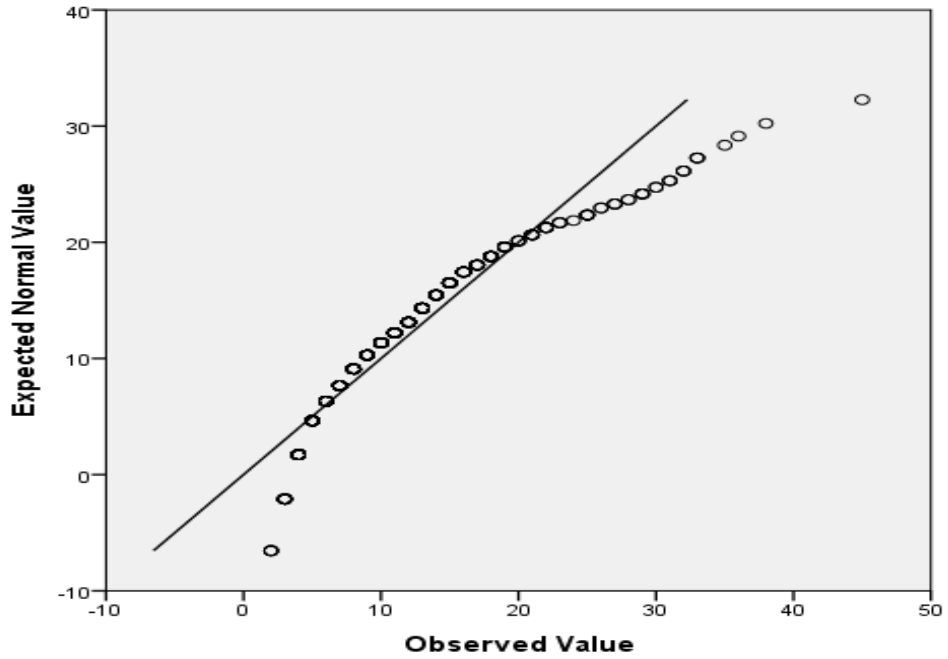


Figure 3: Normal Quantile-Quantile Plot of Waiting Time

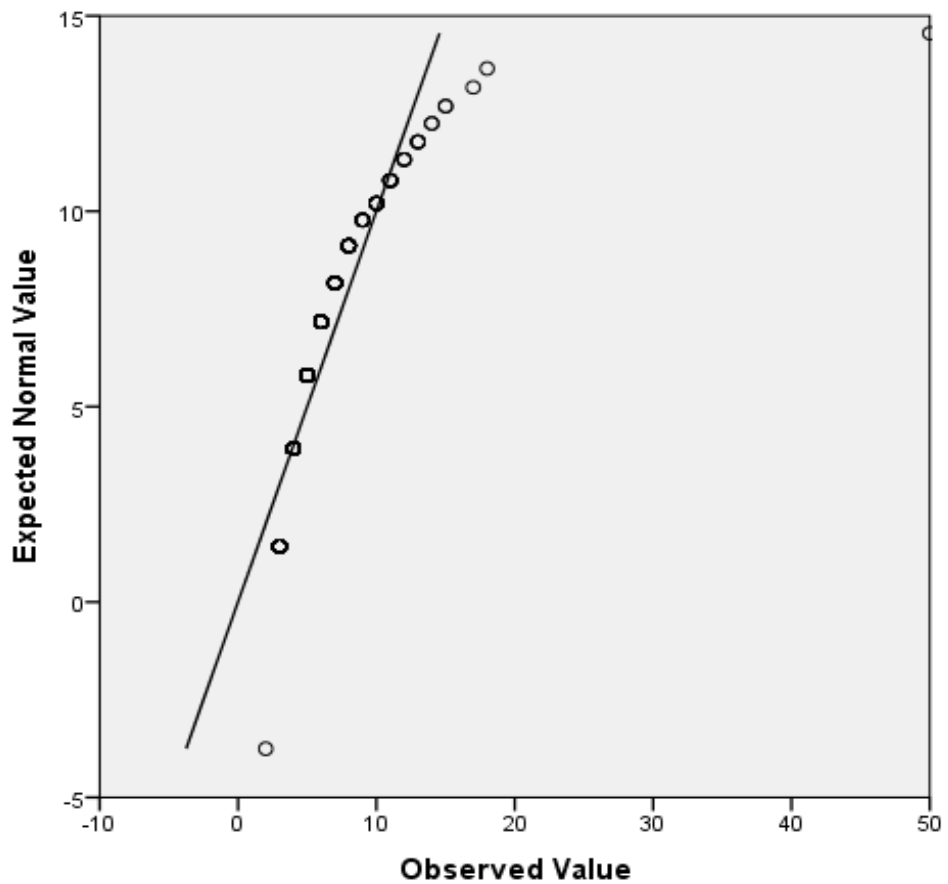


Figure 4: Normal Quantile-Quantile Plot of Service Time

The histograms in Figure 1 and Figure 2 are skewed to the right which suggest that the service time of patients has a distribution that belongs to the gamma family. The normal quantile-quantile in Figure 3 and Figure 4 is used to check if a set of data plausibly came from a theoretical distribution. If both sets of quantiles came from the same distribution, the points forming a line should be roughly straight. The straight line in the plot represents the perfectly normal distribution.

Let S be a random variable representing the service time (in minutes) of patients. An exponential distribution is fitted to the data as follows:

3.1 Exponential Distribution

$H_0 : S \sim \exp(\mu)$ that is service time has an exponential distribution.

$H_1 : S$ does not $\sim \exp(\mu)$ that is service time does not have an exponential distribution.

The probability density function for the exponential distribution is

$$f(s) = \mu e^{-\mu s}, \mu > 0.$$

where

$$\mu = \text{mean service time} = 5.40 \text{ minutes.}$$

The mean of exponential distribution is $\frac{1}{\mu}$, therefore

$$\frac{1}{\mu} = 5.40$$

$$\mu = 0.1852$$

$$P(a < s < b) = \int_a^b 0.1852e^{-0.1852s} ds = e^{-0.1852a} - e^{-0.1852b}$$

3.2 Computation of Expected Frequencies for Exponential Distribution

To determine the number of classes to use and the interval, Sturge’s rule was employed. The rule states that $k = 1 + 3.3 \log n$ where k and n are the number of classes and observations respectively.

Table 2: Frequency Table for Exponential Distribution

Interval Service Time(mins)	Observed Frequency (O _i)	Probability f(s)	Expected Frequency (O _i)
0 < t ≤ 5	358	0.6039	324.2800
6 < t ≤ 10	155	0.1723	92.4800
11 < t ≤ 15	21	0.0682	36.6400
16 < t ≤ 20	2	0.0270	14.5100
> 21	1	0.0205	10.9900

$$X^2 = 72.29$$

$$\chi^2_{k-p-1, \alpha} = \chi^2_{5-1-1, 0.05} = \chi^2_{3, 0.05} = 7.81$$

Since $X^2 = 72.29 > \chi^2_{3, 0.05} = 7.81$, we reject H_0 at 0.05 level of significance and conclude that the service time of patients does not have an exponential distribution.

$H_0 : W \sim \exp(\mu)$ that is waiting time has an exponential distribution.

$H_1 : W$ does not $\sim \exp(\mu)$ that is waiting time does not have an exponential distribution.

The probability density function for the exponential distribution is

$$f(w) = \mu e^{-\mu w}, \mu > 0.$$

where

μ = mean waiting time = 11.54 minutes.

The mean of exponential distribution is $\frac{1}{\mu}$, therefore

$$\frac{1}{\mu} = 11.54$$

$$\mu = 0.087$$

$$P(a < w < b) = \int_a^b 0.087e^{-0.087w} \\ = e^{-0.087a} - e^{-0.087b}$$

In a similar way, the computation for that waiting time is shown below.

To determine the number of classes to use and the interval, Sturge’s rule was employed. The rules states that $k = 1 + 3.3 \log n$ where k and n are the number of classes and observations respectively.

Table 3: Frequency Table for Exponential Distribution

Waiting Time(mins)	Observed Frequency (O _i)	Probability f(w)	Expected Frequency (O _i)
0 < t ≤ 5	107	0.3527	189.4200
6 < t ≤ 10	170	0.1744	93.6400
11 < t ≤ 15	150	0.1129	60.6100
16 < t ≤ 20	57	0.0731	39.2300
21 < t ≤ 25	27	0.0473	25.3900
26 < t ≤ 30	13	0.0306	16.4400
31 < t ≤ 35	10	0.0198	10.6400
> 36	3	0.0436	23.4300

$X^2 = 256.67$

$$\chi^2_{k-p-1, \alpha} = \chi^2_{8-1-1, 0.05} = 6, 0.05 = 12.59$$

Since $X^2 = 256.67 > \chi^2_{3, 0.05} = 7.81$, we reject H_0 at 0.05 level of significance and conclude that the waiting time of patients does not have an exponential distribution.

4.0 CONCLUSION

In this paper, data on 537 patients were collected and analyzed. The mean waiting time was found to be 11.54 minutes while the mean service time was found to be 5.40 minutes. The service time and waiting time of patients does not have an exponential distribution.

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REFERENCES

- Abdellatief, W., & Abdelatey, A. (2022). Analysis of Coronavirus Patients Flow in Hospitals: An Application of Queuing Theory. *IJCI. International Journal of Computers and Information*, 9(1), 46-59.
- Adeniran, A., Burodo, M. S., & Suleiman, S. (2022). Application of Queuing Theory and Management of Waiting Time Using Multiple Server Model: Empirical Evidence From Ahmadu Bello University Teaching Hospital, Zaria, Kaduna State, Nigeria.
- Average ER. Reported by Associated Press; 2008. [Accessed on 7/4/110]. Waiting Time Nears 1 h, CDC Says. Available at: www.oregonlive.com/kiddo/index.../average_er_waiting_time_nears_1h Google
- Aziati, A. N., & Hamdan, N. S. B. (2018, March). Application of queuing theory model and simulation to patient flow at the outpatient department. In *Proceedings of the International Conference on Industrial Engineering and Operations Management Bandung, Indonesia*.
- Bamgboye EO, Erinoso HO, Ogunlesi AO (1994), The waiting time at the children's emergency room, University College Hospital, Ibadan. *Nig J Pediatr*. 19:9-14.
- Dansky KH & Miles J (1997), Patient satisfaction with ambulatory healthcare services: Waiting time and filling time. *Hosp Health Serv Adm*. 142:165-77.
- Dos Santos LM, Stewart G & Rosenberg NM (1994), Pediatric emergency department walk- outs. *Pediatr Emerg Care*.; 10:76-8.
- Huang XM (1994), Patient attitude towards waiting in an outpatient clinic and its applications. *Health Serv Manage Res*.7:2-8.
- Majlesinasab, N., Maleki, M., & Nikbakhsh, E. (2022). Performance evaluation of an EMS system using queuing theory and location analysis: A case study. *The American Journal of Emergency Medicine*, 51, 32-45.
- Mtonga, K., Gatera, A., Jayavel, K., Nyirenda, M., & Kumaran, S. (2022). Adaptive staff scheduling at Outpatient Department of Ntaja Health Center in Malawi-A queuing theory application. *Journal of Public Health Research*.
- Nkeiruka A, Sabo B & Oyefabi MO (2013). Application of queuing theory to patient satisfaction at a tertiary hospital in Nigeria. *Niger Med J* 54(1):64-67.doi:10.4103/0300-1652.108902.
- Oche MO & Adamu H (2013). Determinants of Patient Waiting Time in the General Outpatient Department of a Tertiary Health Institution in North Western Nigeria *Ann Med health Sci Res*. Oct -Dec;3(4): 588-592. Doi:10.4103/2141-9248.122123.
- Ofilli AN & Ofowve CE (2005), Patient's Assessment of Efficiency of Services at a Teaching Hospital in a Developing Country. *Ann Afr Med*. 4:150-3.
- Ofilli A.N & Ofowve C.E (2007). What Patients Want: A Content Analysis of Key Qualities that influence Patient Satisfaction. *J Med Pract Manage*. 22:255-61.
- Ogunlade TO (2019). Application of Queuing Theory to a Bank's Automated Teller Machine (ATM) Service Optimization. *Mathematics Letters*.Vol.5, No. 1,pp.8-12.doi:10.11648/j.ml.20190501.12.
- Oladimeji, A. S., & Ibidoja, O. J. (2020). The Distribution of Service Time of Patients. *Journal of*

Reliability and Statistical Studies, 61-72.

Shanmugasundaram S & Umarani P (2015). Queuing theory applied in our day to day life. *International Journal of Scientific and Engineering Research*, ISSN 2229-5518.

Suleiman, S., Burodo, M. S., & Ahmed, Z. (2022). An Application of Single and Multi-server Exponential Queuing Model in Some Selected Hospitals of the North-Western Nigeria. *Asian Journal of Probability and Statistics*, 16(2), 1-9.

O'Malley MS, Fletcher SW, Fletcher RH & Earp JA (1983), Measuring Patient Waiting Time in a Practice Setting: A comparison of Methods. *J Ambul Care Manage.*6:20–7.

Singh H., Haq ED & Mustapha N (1999), Patients' Perception and Satisfaction with Health Care Professionals at Primary Care Facilities in Trinidad and Tobago. *Bull World Health Organ.* 77:356–60.