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## Mathematical Modeling and Optimal Control of Intervention Strategies for A Banditry Model

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### ABSTRACT

This paper presents an optimal control of intervention strategies for the menace of Banditry taking into account media campaign against Banditry  $u_2(t)$ , rehabilitation of Bandits detainees  $u_5(t)$ , and use of military force against Banditry  $u_6(t)$ , as control strategies. The Banditry free equilibrium, Banditry present equilibrium and the basic reproduction number of Banditry  $R_{0B}$  were obtained. The stability analysis results suggest that the Banditry free equilibrium is locally asymptotically stable when  $R_{0B} < 1$  and otherwise when  $R_{0B} > 1$ . The Boko Haram presence equilibrium is globally asymptotically stable when  $R_{0B} > 1$  and unstable if  $R_{0B} < 1$ . We used the three control strategies and updated the Banditry menace model. The optimal control issue was resolved using Pontryagin's Maximum Principle (PMP). It was discovered that there is a significant decrease in the population of Bandits and increase in the number of rehabilitated Bandits and detained Bandits when the control measures are implemented compared to the case without control. We solved the optimality control using a forward-backward sweep strategy implemented in MATLAB for numerical simulation. Additionally, we saw that the number of people detained fluctuates as the number of people receiving rehabilitation rises. We argue that in order to lessen or completely erase the menace caused by Bandits in society, the government should fund media campaigns and rehabilitation initiatives.

**Keywords:** Banditry; Mathematical Modeling; Rehabilitation; Optimal control problem.

## 1.0 INTRODUCTION

### 1.1 Introduction of the Problem

According to Walker, (2012), a bandit is a robber or armed thief that engages in banditry by attacking others while traveling in quest of money and occasionally other valuable objects. Banditry is an organized crime committed by outlaws utilizing threats or violence. In the northwest of Nigeria, armed banditry has become a significant security issue, terrifying local communities, killing and displacing thousands, impeding socioeconomic development, and endangering democratic governance.

We found it worthy to carry out this research in order to search for the means to curtail the activities

of banditry which happens to be rampant in our societies. Some control strategies were applied in controlling banditry with the intention to trace the most effective ones to so that they can be adopted. It was done through seeking the opinions of some other scholars on banditry.

Bandits target rural residents, often ambushing drivers and killing those resisting kidnapping or refusing ransom. Banditry is a violent crime involving armed robberies and threats. In northern Nigerian states, over 21 million people live in areas with banditry crimes. The discovery of gold mines and illegal miners' activities have worsened armed

bandits' presence in the northwest (Bandits, 2021 & Onuoha, 2014).

There are six main causes cited for why people (youths) join the deadly gang (Banditry): unemployment and poverty, challenging child rearing, high levels of illiteracy, excessive military responses, and widespread corruption. The reasons why young people join terrorism (Banditry) include peer pressure, family and ethnic pressure, marriage, dissatisfaction with the government, a desire for status and prestige, a lack of education, a desire to belong to a group, a desire to make money, a desire to be respected, and other factors (Nadama, 2019) and (Onuoha, 2014).

This essay presents a mathematical model of a socioeconomically stratified society prone to crime. It examines the impact of a non-zero crime rate on a society free of criminals. The model defines a criminal-prone society as unstable under minor changes in socioeconomic conditions. The essay compares two strategies to control crime: enhancing police efficiency or reducing criminal appeal or promoting social classes at risk. (Nuño *et al*, 2011) and (Gambo & Ibrahim, 2020).

A mathematical model examines the dynamics of drug misuse and banditry in the population, determining stable states and assessing reproduction rates. The characteristic equation

### 1.2 Importance to the Problem

It was anticipated that the expansion of the model by Gambo and Ibrahim (2020), would help to reduce the number of people recruited into banditry, hence decreasing the damage they inflicted on people and property. It will help the government to choose the best control strategy to end the threat of banditry in the societies.

In accordance with their area of concern, non-governmental organizations (NGOs) may also utilise the research's findings. They can apply the approach from this research to focus on Bandits.

Multinational enterprises are required in order to cooperatively combat an insurgency across multiple nations. The findings of this study will also help academic institutions decide what control

shows a locally asymptotically stable equilibrium at basic reproduction numbers less than unity, while the center manifold theory of bifurcation reveals a locally asymptotically stable equilibrium at higher reproduction numbers. Bifurcation analysis at the threshold,  $R_0 = 1$  is investigated. Furthermore, it is demonstrated that the banditry and drug abuse model's local asymptotic dynamics display backward bifurcation. Additionally, a sensitivity analysis is carried out to look at how the model settings affect the basic reproduction number of the drug misuse and banditry threat and how that affects the propagation of the threat (Odinkala 2018) and (Olaniyi & Obabiyi, 2013).

Armed banditry and kidnapping have increased in Nigeria's North-West states, causing significant negative consequences on people's lives and property. A study in Katsina state found that armed banditry negatively impacts poverty, unemployment, food security, education, health, income, and overall standard of living. The study suggests tight border security patrols, pro-active measures, universal basic education, skills acquisition centers, and establishing Fulani settlements in under-govern forests. It also calls for adequate farming inputs, food security, and a focus on reviving agricultural production (Muhammad & Bashir, 2022).

strategies to implement for the curtailing the banditry, as well as their lethal operations, to a minimum.

### 1.3 Literature Review

A bandit organization treats the residents of the North Western region similarly. On bicycles equipped with guns, they also attack innocent people and law enforcement officers. Using whatever kind of transportation at their disposal, they attack and cause mayhem. Nigerians and even the international community have grave worries regarding the cycle of violence committed by the Islamist group Boko Haram and Bandits against Nigerians (Ademola, 2021) and (Okpaga *et al*, 2012).

Bandits are also made up of angry young adults, unemployed high school and university graduates,

and orphans, mostly but not solely from northern Nigeria. They also include some wealthy, intelligent, and well-known individuals. Bank robberies, ransom from kidnappings, stop and searches on roadways, cattle rustling, etc. are the main ways that bandits get money. They also demand money from locals in the regions they control and from those they have intimidated into staying away from them (Nadama, 2019) and (Onuoha, 2014).

The study analyzes global health and social issues of illegal narcotics and banditry, focusing on their stability and dynamics. It shows a supercritical bifurcation property and global asymptotically stable equilibria with and without banditry due to the creation of appropriate Lyapunov function. Sensitivity analysis and numerical simulations confirm the findings. Control measures are needed to decrease illegal narcotic use and reduce banditry burden. Akanni & Abidemi, 2023).

## 2.0 METHODS

### 2.1 Model formulation

The modified model sub-divides the population at time  $t$  into five sub-population, these are

susceptible  $S(t)$ , Bandits  $B(t)$ , detention centre for Bandits  $D_T^B(t)$ , rehabilitated Bandits  $R_E^B(t)$ , and Military forces  $M_F(t)$  so that,

$$N(t) = S(t) + B(t) + D_T^B(t) + R_E^B(t) + M_F(t)$$

These assumptions were made during the formulation of the model:

- Military forces (men) can arrest and detain Bandits
- Detainees can be rehabilitated and sent to a vulnerable population.
- As a result of combat and too much casualties due to combat by military, individuals may surrender and move willingly to Bandits rehabilitation centre.
- Stubborn Bandits are moved to detention centre and latter to rehabilitation centre after excessive torturing.
- Every segment of the model has a natural death rate.
- Through contact, susceptible populations can be transformed into Bandits.
- Military action against Banditry force the Bandits to flee to prison centers

Using the above assumptions in mind, the model diagram in Figure 1 was constructed and the system of differential equations in 1 were obtained:

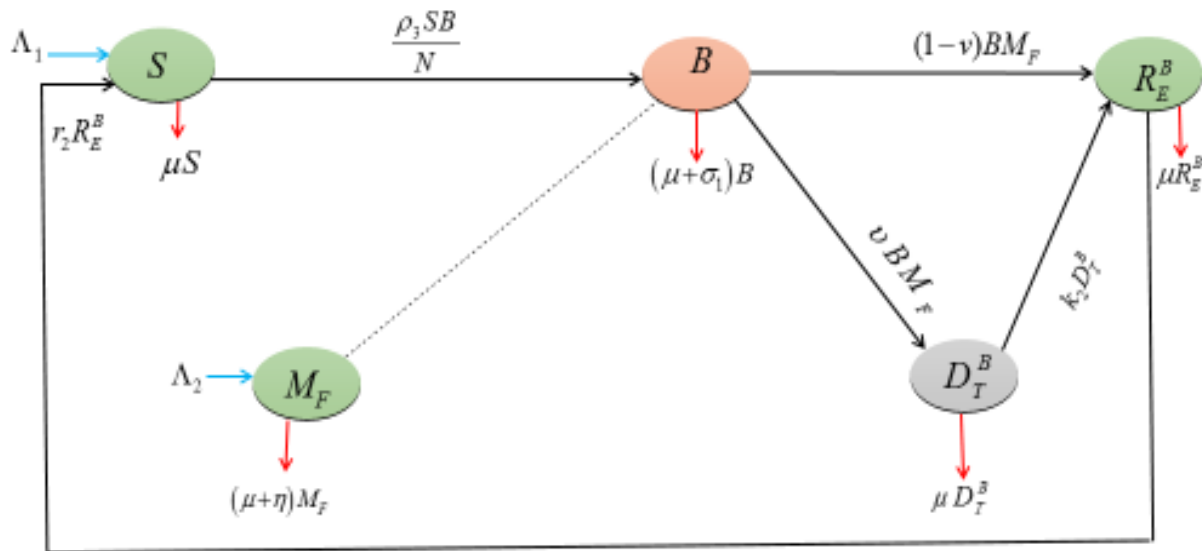


Figure 1: Schematic diagram of Banditry menace with constant controls.

$$\left\{ \begin{array}{l} \frac{dS}{dt} = \Lambda_1 + r_2 R_E^B - \frac{\rho_3 SB}{N} - \mu S \\ \frac{dB}{dt} = \frac{\rho_3 SB}{N} - \nu BM_F - (1-\nu)BM_F - (\mu + \sigma_1)B \\ \frac{dD_T^B}{dt} = \nu BM_F - k_2 D_T^B - \mu D_T^B \\ \frac{dR_E^B}{dt} = (1-\nu)BM_F + k_2 D_T^B - r_2 R_E^B - \mu R_E^B \\ \frac{dM_F}{dt} = \Lambda_2 - (\mu + \eta)M_F \end{array} \right. \quad (1)$$

with the initial conditions

$$\left\{ \begin{array}{l} S(0) = S_0(0) \geq 0, B(0) = B_0(0) \geq 0, D_T^B(0) = D_{T0}^B(0) \geq 0, R_E^B(0) = R_{E0}^B(0) \geq 0, \\ M_F(0) = M_{F0}(0) \geq 0 \end{array} \right. \quad (2)$$

The recruitment rate  $\Lambda_1$  into susceptible population  $S(t)$ , at time  $t$  is done by birth or immigration. The population is also increased by

$$\frac{dS}{dt} = \Lambda_1 + r_2 R_E^B - \frac{\rho_3 SB}{N} - \mu S \quad (3)$$

The population is reduced by adopting Banditry ideologies and move to Banditry sector by the quantity  $\frac{\rho_3 SB}{N}$ . The population further reduced by the natural death rate  $\mu$ . Therefore

$$\frac{dB}{dt} = \frac{\rho_3 SB}{N} - \nu BM_F - (1-\nu)BM_F - (\mu + \sigma_1)B. \quad (4)$$

The population of detained Bandits is increased by military attack on the individuals with mixed ideology which forced them to progress to Bandit detention facilities at the rate  $\theta_1$ . The military attack on Bandits also made them to progress to Bandit detention facility at the rate  $\nu$ . The population here reduced by the rehabilitation of Bandits and move to Bandits rehabilitation centre at the rate  $k_2$ . Furthermore, the population is reduced due to natural death at the rate  $\mu$ , so that

$$\frac{dD_T^B}{dt} = \nu BM_F - k_2 D_T^B - \mu D_T^B. \quad (5)$$

the incoming of rehabilitated members Bandits,  $r_2 R_E^B$  from their compartment. Thus

The population is increased by the number of Bandits who willingly surrendered and moved to the Bandits rehabilitation centre at the rate  $(1-\nu)$ . The population here reduced due to progression of rehabilitated individuals to the susceptible population at the rate  $r_2$ . The population also reduced due to natural death at the rate  $\mu$ . Thus

$$\frac{dR_E^B}{dt} = (1-\nu)BM_F + k_2 D_T^B - r_2 R_E^B - \mu R_E^B. \quad (6)$$

The population of military forces is increased by government reinforcement of military personnel and other defensive weapons at the rate  $\Lambda_2$ . The population here is reduced due to natural death  $\mu$  and killing of military men by bandits at the rate  $\eta$ .

$$\frac{dM_F}{dt} = \Lambda_2 - (\mu + \eta)M_F. \quad (7)$$

The variables, parameters and descriptions were given in table 1 and 3.

**Table 1: Variables used in the model**

Variables	Description
$S(t)$	Population of susceptible individuals at time $t$ .
$B(t)$	Population of Bandits at time $t$ .
$D_T^B(t)$	Detention centre for Bandits at time $t$ .
$R_E^B(t)$	Rehabilitated Bandits individuals at time $t$ .
$M_F(t)$	Number of military forces at time $t$ .

## 2.2 Basic Properties of the Model

In this section, we prove the basic properties of the Banditry model 1.

**Theorem 1.** Let the initial solution set be  $\{S_0 \geq 0, B_0 \geq 0, D_{T0}^B \geq 0, R_{E0}^B \geq 0, M_{F0} \geq 0\} \in R_+^5$ ,

then the solution set  $\{S(t), B(t), D_T^B(t), R_E^B(t), M_F(t)\}$  is positive for all time  $t > 0$

**Proof.** Since the first equation of the model (1) is

$$\frac{dS}{dt} = \Lambda_1 + r_2 R_E^B - \frac{\rho_3 SB}{N} - \mu S,$$

then,

$$\frac{dS}{dt} \geq -\left(\frac{\rho_3 B}{N} + \mu\right)S \quad (8)$$

It resulted to

$$\frac{dS}{dt} \geq -\left(\frac{\lambda}{N} + \mu\right)S \quad (9)$$

where

$$\lambda = \rho_3 B. \quad (10)$$

Solving equation (9) by the method of separating the variables, integrating the result and taking the exponential, we have

$$S(t) \geq ke^{\int -(\frac{\lambda}{N} + \mu) dt} \quad (11)$$

where

$$k = e^C \quad (12)$$

Applying the initial condition at  $t = 0$

$$S(0) \geq k \quad (13)$$

Substituting (13) into equation (11), we have

$$S(t) \geq S(0)e^{\int -(\frac{\lambda}{N} + \mu) dt} > 0.$$

For all  $t \geq 0$ .

Similar technique can be applied to formulate that  $\{B(t), D_T^B(t), R_E^B(t), M_F(t)\}$  are positive for all  $t > 0$ .

**Theorem 2.** The solutions of the model (1) with initial conditions as in Theorem 1 satisfy

$$\{S > 0, B > 0, D_T^B > 0, R_E^B > 0, M_F > 0\}$$

for all  $t \geq 0$ . The region  $\Omega^* \in R_+^5$  is positively invariant and attracting with respect to the model (1)

$$N = S + B + D_T^B + R_E^B + M_F \quad (14)$$

such that  $\Omega = \Omega^* \in R_+^5$

with

$$\Omega^* = \left\{ (S, B, D_T^B, R_E^B, M_F) \in R_+^5 : S + B + D_T^B + R_E^B + M_F \leq \frac{\Lambda_1}{\mu} \right\} \quad (15)$$

*Proof.* Since  $\frac{dN}{dt} = \frac{d}{dt}(S + B + D_T^B + R_E^B + M_F)$  (16)

then,

$$\frac{dN}{dt} = \frac{dS}{dt} + \frac{dB}{dt} + \frac{dD_T^B}{dt} + \frac{dR_E^B}{dt} + \frac{dM_F}{dt} \quad (17)$$

Substituting model (1) into (17) and simplifying, we get

$$\left\{ \frac{dN}{dt} = \Lambda_1 + \Lambda_2 - \mu(S + B + D_T^B + R_E^B + M_F) - (\sigma_1 B + \eta M_F) \right\} \quad (18)$$

Using the comparison theorem on equation (18), it becomes

$$\left\{ \frac{dN}{dt} = \Lambda_1 + \Lambda_2 - \mu(S + B + D_T^B + R_E^B + M_F) \right\} \quad (19)$$

Equation (19) becomes

$$\frac{dN}{dt} \leq \Lambda_1 + \Lambda_2 - \mu N \quad (20)$$

At  $\frac{dN}{dt} = 0$ , equation (23) becomes

$$N = \frac{\Lambda_1 + \Lambda_2}{\mu} \quad (21)$$

Using separation of variables technique on (20), integrating both sides of the result and multiply both sides of the result by  $(-\mu)$ , we obtain

$$\ln(\Lambda_1 + \Lambda_2 - \mu N) \leq -\mu(t + C) \quad (22)$$

Taking the exponent of both sides of (22), we have

$$\Lambda_1 + \Lambda_2 - \mu N \leq k e^{-\mu t} \quad (23)$$

where  $k = e^{-\mu C}$  is taken to be constant in (23).

By applying the initial condition  $t = 0$ ,

$$\Lambda_1 + \Lambda_2 - \mu N(0) = k \quad (24)$$

Substituting (24) into (23), we obtain

$$\Lambda_1 + \Lambda_2 - \mu N(t) \leq (\Lambda_1 + \Lambda_2 - \mu N(0)) e^{-\mu t} \quad (25)$$

Solving for  $N$  in equation (25), we have

$$N(t) \leq \frac{\Lambda_1 + \Lambda_2 - (\Lambda_1 + \Lambda_2 - \mu N(0)) e^{-\mu t}}{\mu} \quad (26)$$

As  $t \rightarrow \infty$ ,  $e^{-\mu t} \rightarrow 0$  in (26), the population size  $N \rightarrow \frac{\Lambda_1 + \Lambda_2}{\mu}$ , which implies that

$$0 \leq N \leq \frac{\Lambda_1 + \Lambda_2}{\mu}. \quad (27)$$

Thus, the feasible solution set of the system of the model (1) is positively invariant in the region  $\Omega$ , hence it is mathematically well-posed. Therefore, it is adequate to study the dynamic values of the model in the region  $\Omega$ .

Table 2: Parameters description and values used

Parameter	Description	Value	Reference
$\Lambda_1$	Constant recruitment rate of susceptible population	0.1	Assumed
$\Lambda_2$	Constant recruitment rate of military.	0.006	Assumed
$\mu$	Natural mortality rate.	0.000034247	Gambo & Ibrahim, (2020)
$\sigma_{n=1,2}$	Death rate due to military attack on Bandits	0.035	Momoh <i>etal</i> (2021).
$\eta$	Death rate of military men caused by Bandits.	0.02	Mebratie & Dawed, (2021)
$k_2$	Rate at which detained Bandits moved to rehabilitation compartment	0.4	Assumed
$\rho_3$	Transfer rate of susceptible population to Bandit	0.25	Momoh <i>etal</i> (2021)
$r_2$	Transfer rate of rehabilitated Bandits to susceptible compartment.	0.5	González-Parra, <i>et al</i> (2018).
$\mathcal{U}$	The rate at which stubborn Bandits moved to Bandit detention centre	0.00004	Mebratie & Dawed, (2021)
$V$	The rate at which stubborn Bandits moved to Bandit detention centre	25	Assumed
$X_1$	Positive weight to balance factor, $u_2$	50	Assumed
$X_2$	Positive weight to balance factor, $u_5$	65	Assumed
$X_3$	Positive weight to balance factor, $u_6$	10	Assumed

### 3.0 MODEL ANALYSIS

This section identifies the equilibrium points of model (1) and verifies the type of stability that model (1) displays

#### 3.1 Equilibrium points

Setting system (1) to zero yields the equilibrium points derived by model (1) as



$$\begin{cases} 0 = \Lambda_1 + r_2 R_E^B - \frac{\rho_3 SB}{N} - \mu S \\ 0 = \frac{\rho_3 SB}{N} - \nu BM_F - (1-\nu)BM_F - (\mu + \sigma_1)B \\ 0 = \nu BM_F - k_2 D_T^B - \mu D_T^B \\ 0 = (1-\nu)BM_F + k_2 D_T^B - r_2 R_E^B - \mu R_E^B \\ 0 = \Lambda_2 - (\mu + \eta)M_F \end{cases} \quad (28)$$

Parameters	Description
$\Lambda_1$	Constant recruitment rate of susceptible population.
$\Lambda_2$	Constant recruitment rate of military.
$\mu$	Natural mortality rate.
$\sigma_1$	Death rate due to military attack on Bandits
$\eta$	Death rate of military men caused by Bandits.
$k_2$	Rate at which detained Bandits moved to rehabilitation compartment.
$\rho_3$	Transfer rate of susceptible population to Bandit.
$r_2$	Transfer rate of rehabilitated Bandits to susceptible compartment.
$\nu$	The rate at which stubborn Bandits moved to Bandit detention centre.
$(1-\nu)$	The rate at which surrendered Bandits moved to Bandits rehabilitation centre

### 3.1.1 Banditry free equilibrium point, $E_{0B}$

The Banditry free equilibrium of the model (1), denoted by  $E_B^0$  is given by

$$E_B^0 = (S^0, B^0, D_T^{B0}, R_E^{B0}, M_F^0) = \left( \frac{\Lambda_1}{\mu}, 0, 0, 0, \frac{\Lambda_2}{\mu + \eta} \right) \quad (29)$$

### 3.1.2 The existence of banditry presence equilibrium point, $E_B^*$

The Banditry free equilibrium of the model (1), denoted by  $E_B^*$  is given by

$$E_B^* = (S^*, B^*, D_T^{B*}, R_E^{B*}, M_F^*) = \left( \frac{N \left[ \Lambda_1 + r_2 (1-\nu) B^* \Lambda_2 + (\mu + \eta) k_2 D_T^{B*} \right]}{(\mu + k_2)(\mu + \eta)(\rho_3 B^* + N\mu)}, \frac{(\mu + k_2)(\mu + \eta) D_T^{B*}}{\nu \Lambda_2}, \frac{\nu B^* \Lambda_2}{(\mu + k_2)(\mu + \eta)}, \right. \\ \left. \frac{(1-\nu) B^* \Lambda_2 + k_2 D_T^{B*}}{(\mu + k_2)(\mu + \eta)}, \frac{\Lambda_2}{(\mu + \eta)} \right) \quad (30)$$

## 3.2 Basic reproduction number of banditry model

We derived the basic reproduction number,  $R_0$  for model (1) using the approach in Diekmann *et al*



(1990), Driessche & Watmough, (2002) and Driessche, (2017), to get

$$F = \begin{bmatrix} \frac{\rho_3 SB}{N} \\ 0 \end{bmatrix} \quad (31)$$

and

$$V = \begin{bmatrix} (\mu + \sigma_1)B \\ (\mu + k_2)D_T^B \end{bmatrix} \quad (32)$$

where  $F$  and  $V$  represent the matrices that contain new infections and transition terms respectively in model (1). Evaluating the Jacobian matrices of  $F$  and  $V$  at Banditry free equilibrium respectively, we have

$$F = \begin{bmatrix} \frac{\rho_3 \Lambda_1}{\Lambda} & 0 \\ 0 & 0 \end{bmatrix} \quad (33)$$

and

$$V = \begin{bmatrix} \mu + \sigma_1 & 0 \\ 0 & \mu + k_2 \end{bmatrix} \quad (34)$$

Therefore, the basic reproduction number of the model (1), denoted by  $R_{0B} = \rho(FV^{-1})$ ,

where  $\rho$  is the spectral radius of  $FV^{-1}$ , is given

$$\text{by } R_{0B} = \frac{\rho_3 \Lambda_1}{\Lambda(\mu + \sigma_1)} \quad (35)$$

### 3.3 Local stability of banditry equilibrium point

**Theorem 3.** *The Bandit free equilibrium point is locally asymptotically stable if  $R_{0B} < 1$  and unstable if  $R_{0B} > 1$*

*Proof.* The Jacobean matrix for the banditry only model at free equilibrium is given as

$$J_{E_0^B} = \begin{bmatrix} -\mu & 0 & 0 & r_2 & 0 \\ 0 & \frac{\rho_3 \Lambda_1}{\Lambda} - \left[ \frac{\Lambda_2}{(\mu + \eta)} + (\mu + \sigma_1) \right] & 0 & 0 & 0 \\ 0 & \frac{v\Lambda_2}{(\mu + \eta)} & -(\mu + k_2) & 0 & 0 \\ 0 & \frac{(1-v)\Lambda_2}{(\mu + \eta)} & k_2 & -(\mu + r_2) & 0 \\ 0 & 0 & 0 & 0 & -(\mu + \eta) \end{bmatrix} \quad (36)$$

From equation (38), we have  $\lambda_1 = -\mu < 0$ ,  $\lambda_5 = -(\mu + \eta) < 0$ ,

Thus, equation (38) reduces to

$$J_{E_0^B} = \begin{bmatrix} \frac{\rho_3 \Lambda_1}{\Lambda} - \left[ \frac{\Lambda_2}{(\mu + \eta)} + (\mu + \sigma_1) \right] & 0 & 0 \\ \frac{v\Lambda_2}{(\mu + \eta)} & -(\mu + k_2) & 0 \\ \frac{(1-v)\Lambda_2}{(\mu + \eta)} & k_2 & -(\mu + r_2) \end{bmatrix} \quad (37)$$

Furthermore, we also observed from equation (39) that  $\lambda_3 = -(\mu + r_2) < 0$  which reduced equation (39) to

$$J_{E_0^B} = \begin{bmatrix} \frac{\rho_3 \Lambda_1}{\Lambda} - \left[ \frac{\Lambda_2}{(\mu + \eta)} + (\mu + \sigma_1) \right] & 0 \\ \frac{\nu \Lambda_2}{(\mu + \eta)} & -(\mu + k_2) \end{bmatrix} \quad (38)$$

Thus, the system will be asymptotically stable at  $J_{E_0^B}$  if the remaining eigenvalues satisfy the condition of Routh-Hurwitz from the following equation

$$\lambda^2 + B_1 \lambda + B_2 = 0 \quad (39)$$

where 
$$\begin{cases} B_0 = 1 \\ B_1 = A_1 + A_2 \\ B_2 = A_1 A_2 \end{cases} \quad (40)$$

Further expressing  $B_2$  in terms of reproduction number,  $R_{0B}$  we obtain

$$B_2 = A_1 A_2 = \left( \frac{\Lambda_2}{(\mu + \eta)} + (\mu + \sigma_1) - \frac{\rho_3 \Lambda_1}{\Lambda} \right) (\mu + k_2) = A_1 A_2 \left( 1 - \frac{\rho_3 \Lambda_1}{\Lambda(\mu + \sigma_1)} \right) = A_1 A_2 (1 - R_{0B})$$

Applying Routh-Hurwitz criterion implies the roots of polynomial (39) have negative real parts if the coefficient  $B_1 > 0$ ,  $B_2 > 0$ , since  $A_i$ 's are positive. Further, if  $R_{0B} < 1$ , it follows from (39) that  $B_2 > 0$  (Okpaga, *et al.*, 2012). Therefore, all the eigenvalues of the Jacobian matrix  $J_{E_0^B}$  have

negative real parts when  $R_{0B} < 1$  and the Banditry free equilibrium point is locally asymptotically stable. In other words, if one bandit is brought into a population that is vulnerable, he can initiate less than one person into banditry if  $R_{0B} < 1$  and otherwise if  $R_{0B} > 1$ , until the banditry eventually disappears

### 3.4 Banditry presence global stability

We adopt the Lyapunov function to determine the global stability of Banditry.

**Theorem 4.** *If the  $R_{0B} > 1$ , then the Banditry presence equilibrium is globally asymptotically stable.*

*Proof.* Here, we considered the Lyapunov function defined as

$$L(S^*, B^*, R_E^{B*}) = (S^* - S - S^* \ln \frac{S}{S^*}) + (B^* - B - B^* \ln \frac{B}{B^*}) + (R_E^{B*} - R_E - R_E^{B*} \ln \frac{R_E^{B*}}{R_E^B}) \quad (41)$$

Determining the derivative of  $L$  along the solution of the system (41) directly,

$$\frac{dL}{dt} = \left( 1 - \frac{S^*}{S} \right) \frac{dS}{dt} + \left( 1 - \frac{B^*}{B} \right) \frac{dB}{dt} + \left( 1 - \frac{R_E^{B*}}{R_E^B} \right) \frac{dR_E^B}{dt} \quad (42)$$

Replacing the derivatives of the variables with respect to  $t$ , we have

$$\begin{aligned} \frac{dL}{dt} = & \left( 1 - \frac{S^*}{S} \right) \left( \Lambda_1 + r_2 R_E^B - \frac{\rho_3 S B}{N} - \mu S \right) + \left( 1 - \frac{B^*}{B} \right) \left( \frac{\rho_3 S B}{N} - \nu B M_F - (1 - \nu) B M_F - (\mu + \sigma_1) B \right) \\ & + \left( 1 - \frac{R_E^{B*}}{R_E^B} \right) \left( (1 - \nu) B M_F + k_2 D_T^B - r_2 R_E^B - \mu R_E^B \right) \end{aligned} \quad (43)$$

At steady state,

$$\Lambda_1 = \frac{\rho_3 S^* B^*}{N^*} + \mu S^* - r_2 R_E^{B*} \quad (44)$$

Substituting (44) into (43), collecting all the infected class without asterisk (\*) and equating to zero, we have

$$\left\{ \begin{aligned} \frac{dL}{dt} = & (2\mu S^* - \mu S - \frac{\mu S^{*2}}{S}) + \rho_3 S^* B^* - \rho_3 S^* B^* \frac{S^*}{S} + r_2 R_E^{B^*} \frac{S^*}{S} - r_2 R_E^B \frac{S^*}{S} \\ & + \rho_3 S^* B - \rho_3 S B^* + \nu B M_F - \nu B^* M_F (\mu + \sigma_1) B^* - (\mu + \sigma_1) B \\ & + (1 - \nu) M_F B^* - (1 - \nu) M_F B \frac{R_E^{B^*}}{R_E^B} + k_2 D_T^B - k_2 D_T^B \frac{R_E^{B^*}}{R_E^B} + \mu R_E^{B^*} - \mu R_E^B \end{aligned} \right. \quad (45)$$

Now factorizing equation (45), we have

$$\left\{ \begin{aligned} \frac{dL}{dt} = & \mu S^* \left( 2 - \frac{S}{S^*} - \frac{S^*}{S} \right) + \rho_3 S^* B^* \left( 1 - \frac{S^*}{S} \right) + r_2 R_E^{B^*} \frac{S^*}{S} \left( 1 - \frac{R_E^B}{R_E^{B^*}} \right) + \rho_3 S^* B \left( 1 - \frac{B^*}{B} \right) + \nu B^* M_F \left( 1 - \frac{B}{B^*} \right) \\ & + (\mu + \sigma_1) B^* \left( 1 - \frac{B}{B^*} \right) + (1 - \nu) M_F B^* \left( 1 - \frac{B}{B^*} \frac{R_E^{B^*}}{R_E^B} \right) + k_2 D_T^B \left( 1 - \frac{R_E^{B^*}}{R_E^B} \right) + \mu R_E^{B^*} \left( 1 - \frac{R_E^B}{R_E^{B^*}} \right) \end{aligned} \right. \quad (46)$$

Since the geometric mean is less than the arithmetic mean (Maia, 2015), the following inequality from equation (46) holds

$$\frac{S}{S^*} - \frac{S^*}{S} \geq 2, \quad 1 \leq \frac{S^*}{S}, \quad 1 \leq \frac{R_E^B}{R_E^{B^*}}, \quad 1 \leq \frac{B^*}{B}, \quad 1 \leq \frac{B}{B^*} \frac{R_E^{B^*}}{R_E^B},$$

Therefore,  $\frac{dL}{dt} \leq 0$  for  $R_{0B} > 1$  and  $\frac{dL}{dt} = 0$  if and only if  $S = S^*, B = B^*, R_E^B = R_E^{B^*}$

Hence, L is a Lyapunov function on the region of the system which agrees with the LaSalle's Principle, which states that every solution to the equations of the model tends to endemic equilibrium of the model as  $t \rightarrow \infty$  for  $R_{0B} > 1$ . Therefore, the Banditry presence equilibrium point is globally asymptotically stable in the invariant region  $\Omega$  if  $R_{0B} > 1$  (Alouch, *et al*, 2019); (Korobeinikov & Wake, 2002); (Pykh, 2001) and (Shuai & Van Den, 2013). This stated that if a bandit were to happen to be in a potential population, he could recruit additional individuals if  $R_{0B} > 1$ , to join him in committing crimes. As a result, the threat persists as practically everyone eventually engages in banditry (Onuoha, 2014).

### 3.5 Sensitivity Analysis

$$S_{\Lambda_1}^{R_{0B}} = \frac{\partial R_{0B}}{\partial \Lambda_1} \times \frac{\Lambda_1}{R_{0B}} \quad (47)$$

We used the parameter values on table 4.2 to calculate this sensitivity index of Banditry sub-model. The sensitivity indices of the basic reproduction number are calculated in order to determine how important each parameter is in the initiation of Banditry: that is, in the control of Banditry, the parameters that have the greatest influence on the basic reproduction number, as well as intervention strategies that target such parameter, should be used.

We adopted the local sensitivity analysis based on the normalized forward sensitivity index  $R_{0B}$ . The sensitivity index of  $R_{0B}$  with respect the parameters in our model are derived and used it as in equation (49), using a parameter  $\Lambda_1$  of our work

$$S_{\Lambda_1}^{R_{0B}} = \frac{\partial R_{0B}}{\partial \Lambda_1} \times \frac{\Lambda_1}{R_{0B}} = \frac{\partial(\frac{\rho_3 \Lambda_1}{\Lambda(\mu + \sigma_1)})}{\partial \Lambda_1} \times \frac{\Lambda_1}{\frac{\rho_3 \Lambda_1}{\Lambda(\mu + \sigma_1)}} = \frac{\rho_3}{\Lambda(\mu + \sigma_1)} \times \frac{\Lambda_1}{\frac{\rho_3 \Lambda_1}{\Lambda(\mu + \sigma_1)}} = +1 \quad (48)$$

The remaining sensitivity indices were calculated in a similar way. The results are given in table 3.

**Table 3: Sensitivity index for basic reproduction number for Bandits**

Parameter	value	Sensitivity Index
$\Lambda_1$	0.1	+1
$\Lambda_2$	0.006	-1
$\mu$	0.00003427	-0.0007997
$\sigma_1$	0.035	+0.999
$\rho_3$	0.25	+1

It is observed from Table 4.1 that the parameters  $\Lambda_1$ ,  $\sigma_1$  and  $\rho_3$  are having positive indices which means that increasing any one of these leads the raise in the reproduction number of Banditry, consequently, increase the chance of Banditry

### 3.6 Optimal Control Analysis

The optimal control interventions for effective control of the Banditry menace is presented in this section. From the result of the sensitivity analysis carried out, we observed that some parameters in

activities in our societies. On the contrary, the parameters,  $\Lambda_2$  and  $\mu$  have negative indices depicting that increasing one of these parameters while maintaining the others reduces effective reproduction number, hence lowering the activities of Bandits in our communities.

the model have impact on basic reproduction number of Banditry  $R_{0B}$  model

To address it, some controls are incorporated in the model 1 to have

$$\begin{cases} \frac{dS}{dt} = \Lambda_1 + r_2 R_E^B - \frac{\rho_3 SB}{N} - \mu S \\ \frac{dB}{dt} = \frac{\rho_3 SB}{N} - u_6 BM_F - (\mu + \sigma_1) B \\ \frac{dD_T^B}{dt} = u_6 \nu BM_F - u_5 k_2 D_T^B - \mu D_T^B \\ \frac{dR_E^B}{dt} = u_6 (1 - \nu) BM_F + u_5 k_2 D_T^B - r_2 R_E^B - \mu R_E^B \\ \frac{dM_F}{dt} = \Lambda_2 - (\mu + \eta) M_F \end{cases} \quad (49)$$

The steps in Pontryagin's (1962) is adopted to formulate the Objective functional  $J$  for the Banditry menace only model to investigate the optimal level of the effort required to control the Banditry menace. We introduce three control

measures, namely, media campaign against Banditry  $u_2(t)$ , rehabilitation of detained Bandits  $u_5(t)$ , and use of military force against Banditry  $u_6(t)$ . Thus the objective functional  $J$  is

$$J(u_{2,5,6}) = \int_0^{t_f} (V B + \frac{1}{2} X_1 u_2 + \frac{1}{2} X_2 u_5 + \frac{1}{2} X_3 u_6) dt \quad (50)$$

where  $t_f$  is the final time and the coefficient  $V$ ,  $X_1$ ,  $X_2$ , and  $X_3$ , are positive weight to balance the factors. Our major aim is to minimize the number of Bandits while we keep cost of controls  $u_2(t)$ ,  $u_5(t)$ , and  $u_6(t)$  at a very low cost. Thus seek an optimal controls  $u_2^*$ ,  $u_5^*$ , and  $u_6^*$  such that

$$J(u_i^*) = \min \{ J(u_i) : u_i \in U \}, i = 2, 5, 6$$

where  $U$  is the set of all measurable functions defined from  $[0, t_f]$  onto  $[0, 1]$ .

The necessary conditions that an optimal control must satisfy were derived from PMP (Pontryagin *et al*, 1962). and the existence result for optimal control from the adjoint variable of the state variables satisfy the following set of differential equations. This principle converts system (51)- (52) into a problem of minimizing point-wise a Hamiltonian  $H$ , with respect to the controls  $(u_2, u_5, u_6)$ . The Hamiltonian is given by:

$$\begin{aligned} H = & VB + \frac{1}{2} X_1 u_2^2 + \frac{1}{2} X_2 u_5^2 + \frac{1}{2} X_3 u_6^2 \\ & + \lambda_S \left[ \Lambda_1 + r_2 R_E^B - \frac{(1-u_2) \rho_3 S B}{N} - \mu S \right] \\ & + \lambda_B \left[ \frac{(1-u_2) \rho_3 S B}{N} - u_6 B M_F - (\mu + u_6 \sigma_1) B \right] \\ & + \lambda_{D_T^B} \left[ u_6 \nu B M_F - u_5 k_2 D_T^B - \mu D_T^B \right] \\ & + \lambda_{R_T^B} \left[ u_5 k_2 D_T^B - u_6 (1-\nu) B M_F - r_2 R_T^B - \mu R_T^B \right] \\ & + \lambda_{M_F} \left[ \Lambda_2 - (\mu + \eta) M_F \right] \end{aligned} \quad (51)$$

where  $\lambda_S, \lambda_B, \lambda_{D_T^B}, \lambda_{R_T^B}, \lambda_{M_F}$ , are adjoint variables and

**Theorem 5.** Let  $u_2^*, u_5^*$ , and  $u_6^*$  be optimal controls and  $S^*, B^*, D_T^{B*}, R_E^{B*}, M_F^*$ , the solutions of the optimal control problems of (49) and (51) that minimizes  $J(u_2, u_5, u_6)$  over  $U$ , then there exists adjoint variables  $\lambda_S, \lambda_B, \lambda_{D_T^B}, \lambda_{R_T^B}, \lambda_{M_F}$  that satisfies

$$\frac{d\lambda_i}{dt} = - \frac{\partial H}{\partial i} \quad (52)$$

where  $\lambda_i$  for  $i = S, B, D_T^B, R_E^B, M_F$  are the adjoint variables and the controls  $u_2^*, u_5^*$ , and  $u_6^*$  obey the optimal conditions, such that

$$\left. \begin{aligned} u_2^* &= \max \left\{ 0, \min \left( 1, \frac{\rho_3 B^* S^* (\lambda_B - \lambda_S)}{X_1 N^*} \right) \right\} \\ u_5^* &= \max \left\{ 0, \min \left( 1, \frac{k_2 D_T^{B^*} (\lambda_{D_T^B} - \lambda_{R_E^B})}{X_2} \right) \right\} \\ u_6^* &= \max \left\{ 0, \min \left( 1, \frac{B^* M_F^* (\lambda_B - \lambda_{R_E^B}) + \nu B^* M_F^* (\lambda_{R_E^B} - \lambda_{D_T^B}) + \sigma_1 B^* \lambda_B}{X_3} \right) \right\} \end{aligned} \right\} \quad (53)$$

*Proof.* To obtain the differential equations that governs the adjoint variable, we assessed the differential Hamiltonian functions at the optimal control. Hence

$$\left\{ \begin{aligned} \frac{dH}{d\lambda_S} &= \frac{(1-u_2)\rho_3 B}{N} (\lambda_S - \lambda_B) - \mu \lambda_S \\ \frac{dH}{d\lambda_B} &= \frac{(1-u_2)\rho_3 B}{N} (\lambda_S - \lambda_B) + u_6 M_F (\lambda_S - \lambda_{R_E^B}) + u_6 \nu M_F (\lambda_{R_E^B} - \lambda_{D_T^B}) + (\mu + u_6 \sigma_1) \lambda_B - V \\ \frac{dH}{d\lambda_{D_T^B}} &= u_5 k_2 (\lambda_{D_T^B} - \lambda_{R_E^B}) + \mu \lambda_{D_T^B} \\ \frac{dH}{d\lambda_{R_E^B}} &= r_2 (\lambda_{R_E^B} - \lambda_S) + \mu \lambda_{R_E^B} \\ \frac{dH}{d\lambda_{M_F}} &= u_6 B (\lambda_B - \lambda_{R_E^B}) + u_6 \nu B (\lambda_{D_T^B} - \lambda_{R_E^B}) + (\mu + \eta) \lambda_{M_F} \end{aligned} \right. \quad (54)$$

with transversality conditions:

$$\lambda_S(t_f) = \lambda_B(t_f) = \lambda_{D_T^B}(t_f) = \lambda_{R_E^B}(t_f) = \lambda_{M_F}(t_f) = 0 \quad (55)$$

Also, the optimal functions  $u_2^*, u_5^*, u_6^*$  satisfy

$$\frac{\partial H}{\partial u_i} = 0, i = 2, 5, 6.$$

Therefore,

$$u_2^* = \frac{\rho_3 S^* B^*}{X_1 N^*} (\lambda_B - \lambda_S) \quad (56)$$

$$u_5^* = \frac{k_2 D_T^{B^*}}{X_2} (\lambda_{D_T^B} - \lambda_{R_E^B}) \quad (57)$$

$$u_6^* = \frac{B^* M_F^* (\lambda_B - \lambda_{R_E^B}) + \nu B^* M_F^* (\lambda_{R_E^B} - \lambda_{D_T^B}) + \sigma_1 B^* \lambda_B}{X_3} \quad (58)$$

The optimal control functions are given as

$$\begin{cases}
u_2^* = \max \left\{ 0, \min \left( 1, \frac{\rho_3 B^* S^* (\lambda_B - \lambda_S)}{X_1 N^*} \right) \right\} \\
u_5^* = \max \left\{ 0, \min \left( 1, \frac{k_2 D_T^{B^*} (\lambda_{D_T^B} - \lambda_{R_E^B})}{X_2} \right) \right\} \\
u_6^* = \max \left\{ 0, \min \left( 1, \frac{B^* M_F^* (\lambda_B - \lambda_{R_E^B}) + \nu B^* M_F^* (\lambda_{R_E^B} - \lambda_{D_T^B}) + \sigma_1 B^* \lambda_B}{X_3} \right) \right\}
\end{cases} \quad (59)$$

Based on the earlier boundedness of the system (49) and the adjoint system (51), we get the uniqueness of the optimality systems (54), (55) and (59). To ascertain the uniqueness of the optimality system, we considered the length of

the time interval  $[0, t_f]$  (Momoh, *etal* (2021).

Based on the standard central argument taking into account the bounds in the control (Momoh, *etal* (2021) and (Suzanne & John, 2007)., we get

$$u_2^* = \begin{cases} 0 & \text{if } u_2^* \leq 0 \\ u_2^* & \text{if } 0 \leq u_2^* \leq 1 \\ 1 & \text{if } u_2^* \geq 1 \end{cases} \quad u_5^* = \begin{cases} 0 & \text{if } u_5^* \leq 0 \\ u_5^* & \text{if } 0 \leq u_5^* \leq 1 \\ 1 & \text{if } u_5^* \geq 1 \end{cases} \quad u_6^* = \begin{cases} 0 & \text{if } u_6^* \leq 0 \\ u_6^* & \text{if } 0 \leq u_6^* \leq 1 \\ 1 & \text{if } u_6^* \geq 1 \end{cases} \quad (60)$$

#### 4.0 DISCUSSION

The impact of interventions on the proliferation of banditry within a population is discussed in this section. To quantitatively solve the optimal control problem (59) and the adjoint variable (51), we employed the approach used by control (Momoh *etal* (2021) and (Suzanne & John, 2007).

These methods employ the Runge-Kutta forward and backward technique. In order to lower or eradicate the number of bandits, we use the variables and parameter values in Tables 2 and 4. One hundred (100) days have been set aside as

the time frame for the work. The simulations in Figs. 5.5–5.8 illustrate the application of the following intervention strategies:

- 4.1. Strategy A: Media campaign and Rehabilitation of Bandits
- 4.2. Strategy B: Media campaign and use of military force against Bandit
- 4.3. Strategy C: Rehabilitation of Bandits and use of Military forces.
- 4.4. Strategy D: Media campaign, rehabilitation of Bandits and use of military forces.

**Table 4: Initial values for variables used in the model**

variable	Value	Reference
$S(0)$	8000	Muhammad & Bashir, (2022).
$B(0)$	350	Muhammad & Bashir, (2022).
$D_T^B(0)$	100	Gambo & Ibrahim, (2020).
$R_E^B(0)$	80	Muhammad & Bashir, (2022).
$M_F(0)$	500	Assumed



#### 4.1. Strategy A: Media campaign and Rehabilitation of Bandits

When the use of military force is set at zero, intervention strategy A shows the simulation of an optimal control system (49) for the implementation of a media campaign against Banditry ( $u_2$ ) and the rehabilitation of Bandits ( $u_5$ ). We observed from Figure 5.5 (a,b,c) that in the absence of media campaign against Banditry ( $u_2$ ) and the rehabilitation of Bandits ( $u_5$ ), indicated by blue line, the population of Bandits ( $B$ ) and detained Bandits ( $D_T^B$ ) are at top level, but that of rehabilitated Bandits, ( $R_E^B$ ) just below average number. When the method is put into practice (indicated by red line) compared to the scenario without control, we see a drop in the population of Bandits ( $B$ ) and detained Bandits ( $D_T^B$ ) but the population of rehabilitated Bandits, ( $R_E^B$ ) remained unchanged as shown in Figure 5.5(a,b,c) respectively. The control profile for strategy A shown in Figure 5.5 (d), The control curve for the media campaign against banditry remains at the lower bound for the entire duration of the study period while the curve for the rehabilitation of Bandits ( $u_5$ ) remains at the upper bound for the targeted days (100 days) before descending to the lower bound.

#### 4.2. Strategy B: Media campaign and Use of Military Force Against Bandit

Intervention strategy B presents the simulation of optimal control system (49) for implementation of media campaign against Banditry ( $u_2$ ) and the use of Military force ( $u_6$ ) while the use of rehabilitation method ( $u_5$ ) is fixed at zero. It was observed from Figure 5.6 (a,b,c) that in the absence of media campaign against Banditry ( $u_2$ ) and the use of military force ( $u_6$ ) (shown by blue line), the population of Bandits ( $B$ ) and the number of detained Bandits ( $D_T^B$ ) remains at high level while

that of rehabilitated Bandits ( $R_E^B$ ) is very low, respectively. It was observed from Figure 5.6 (c) that there is decline in the population of detained Bandits ( $D_T^B$ ) and sharp increase in the population of rehabilitated bandits, but after 10 days, the rehabilitated bandits declined ( $R_E^B$ ) as the days increase when there is control as indicated in the red line as compared to if there is no control. The increase is probably due to movement of rehabilitated Bandits to susceptible compartment. Figure 5.6 (d) shows the control profile for strategy B where we observed that the control curve for the use of military force raised sharply to 60% implementation and after one day, drop back to 48%. After a day again, it raised sharply to 100% implementation and stay at the upper bound for the targeted days before declining to the lower bound.

#### 4.3. Strategy C: Rehabilitation of Bandits and use of Military Forces

The optimal control system (49) for the implementation of the rehabilitation of Bandits ( $u_5$ ) and the deployment of Military force ( $u_6$ ) is presented in intervention plan C, with the media campaign against banditry ( $u_2$ ) is set at zero. In contrast to the situation where there is no control, as indicated by blue line, it can be seen from Figure 5.7(a) that the population of bandits ( $B$ ) is high, In Figure 5.7(b), the population of detained bandits ( $D_T^B$ ) is minimal while in Figure 5.7(c), there is very scanty rehabilitated Bandits ( $R_E^B$ ). When we employed the strategies using red lines, the population of bandits ( $B$ ) drops sharply as shown in Fig. 5.3(a), but the population of detained bandits ( $D_T^B$ ) and the rehabilitated Bandits ( $R_E^B$ ) increases as depicted in Figure.5.7 (b,c), respectively. Figure 5.7 (d) displays the control profile for strategy C. In our research, we discovered that the control curve for the rehabilitation of Bandits ( $u_5$ ) falls from 45% to 0% implementation and remains at the lower bound throughout the entire study time. After five days, the implementation curve for using of military force ( $u_6$ ) increased from 0% to 100%

and remain there for the entire duration of study period.

#### 4.4. Strategy D: Media Campaign, Rehabilitation of Bandits and use of Military Forces

The execution of a media campaign against Bandits, the rehabilitation of bandits, and the employment of military force are presented in intervention strategy D's simulation of the optimal control system (49). When the strategy is put into practice, we see a sharp increase in the population of rehabilitated bandits and a sharp decrease in the population of detained bandits in Figure 5.8. However, after 10 days, the rehabilitated bandits start to decline as the days go on as opposed to the case where there is no control. Figure 5.8 (d) displays the control profile for strategy D. We discovered that the media campaign against bandits' control curve (red curve) began at 50% and increased to 100% implementation. After staying there for 60 days, it returns to the lower bound. The curve for the rehabilitation of bandits (blue) drops from a 45% implementation to a 0% implementation and remains at the lower bound for the entire study period. The curve for the application of military force (green colour) increased from 1% to 100%, and after 5 days, it remained at the upper bound for 99 days before descending to the lower bound.

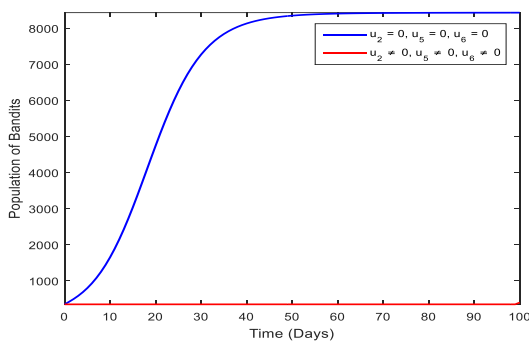


Figure 5.5 (a): Graphical solutions for implementation of media campaign against bandits and rehabilitation of bandits to reduce the population of bandits

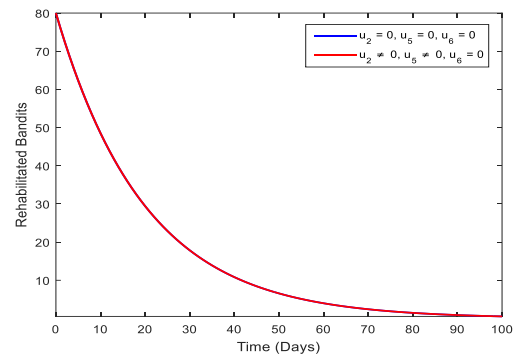


Figure 5.5 (b): Graphical solutions for implementation of media campaign against bandits and rehabilitation of bandits to curtail rehabilitated bandits

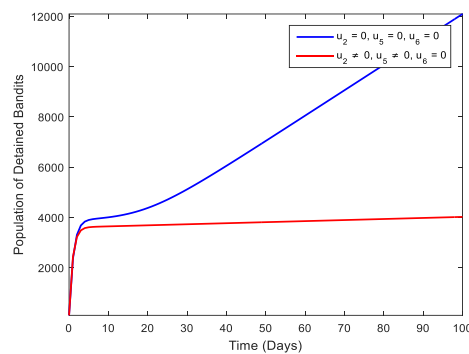


Figure 5.5(c): Graphical solutions of media campaign against bandits and rehabilitation of bandits to reduce the population of detained bandits

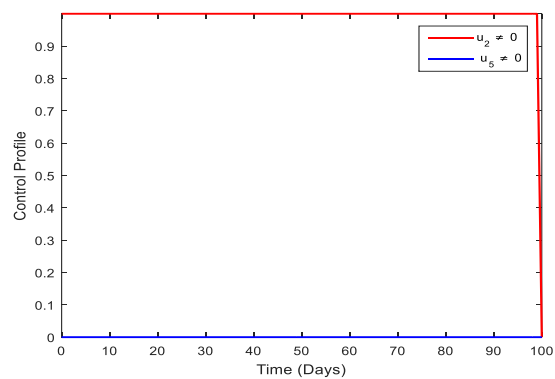
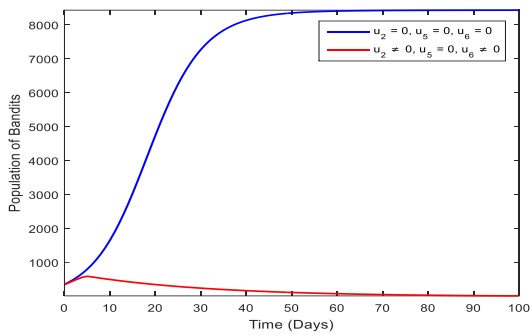
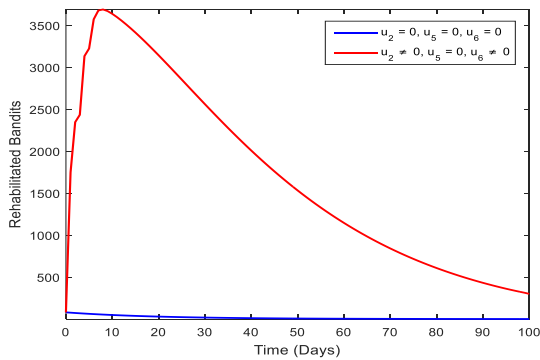


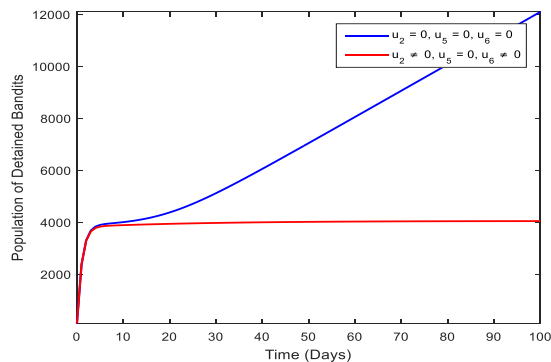
Figure 5.5 (d): The control profile of the control measures



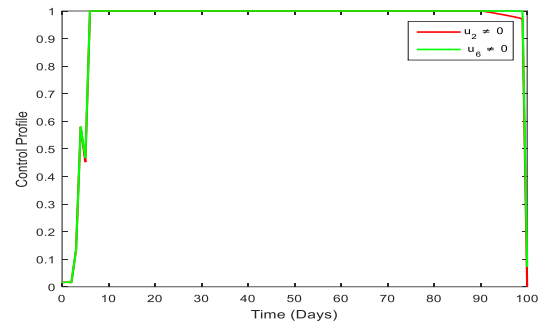
**Figure 5.6 (a):** Graphical solutions for implementation of media campaign against bandits and use of military force on bandits to fight bandits



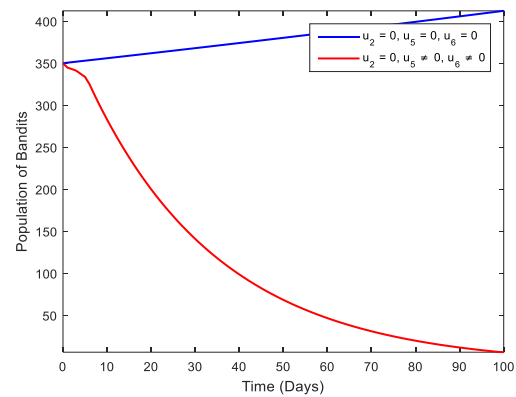
**Figure 5.6 (b):** Graphical solutions for implementation of media campaign against bandits and use of military force on bandits to fight rehabilitated bandits



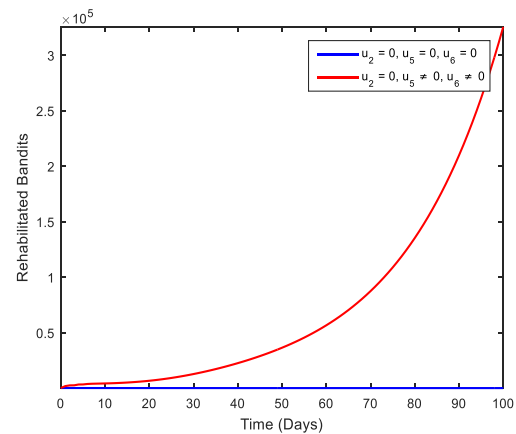
**Figure 5.6 (c):** Graphical solutions for implementation of media campaign against bandits and use of military force on bandits to reduce detained bandits



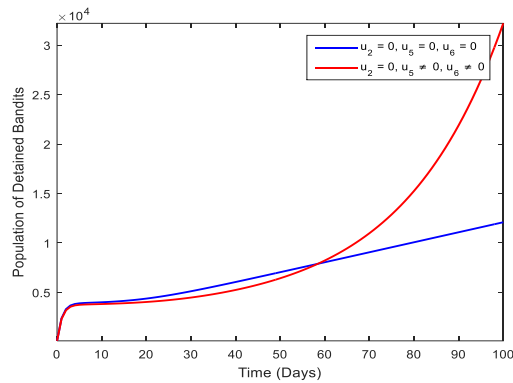
**Figure 5.6 (d):** The control profile of the control measures



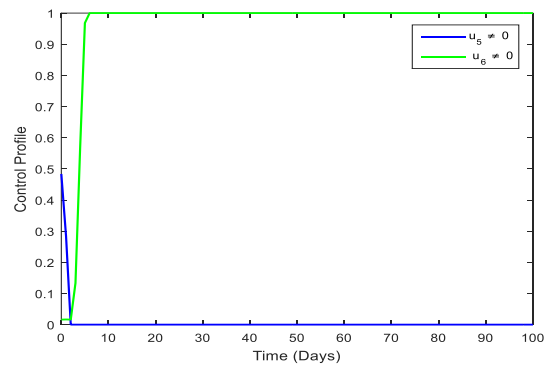
**Figure 5.7 (a):** Graphical solutions for implementation of rehabilitation of Bandits and use of military force to eradicate bandits



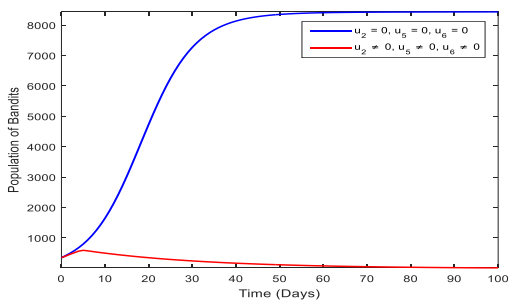
**Figure 5.7 (b):** Graphical solutions for implementation of rehabilitation of Bandits and use of military force to reduce rehabilitated bandits



**Figure 5.7 (c): Graphical solutions for implementation of rehabilitation of Bandits and use of military force to reduce detained bandits**



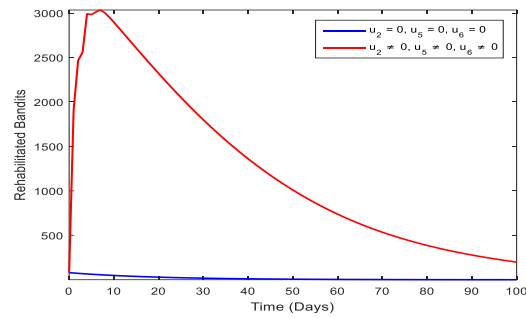
**Figure 5.7 (d): The control profile of the control measures**



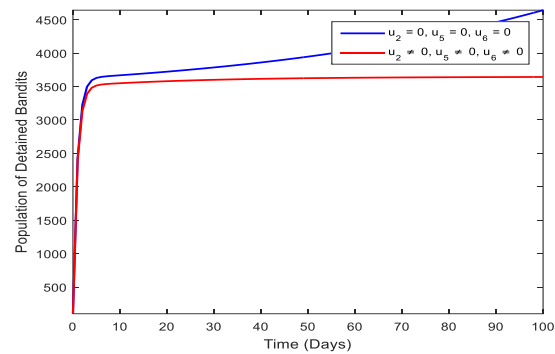
**Figure 5.8 (a): Graphical solutions for implementation of media campaign against bandits, rehabilitation of bandits and use of military force to fight banditry**

## 5.0 CONCLUSION

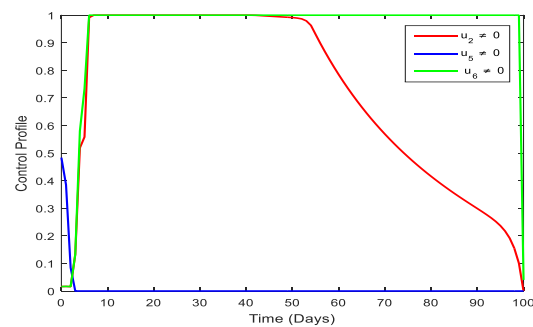
The research work proposed and examined a mathematical model and the optimal control for preventing the spread of Banditry menace activities in our societies, including media campaigns, rehabilitation programs, detention, and the use of military force against Banditry. By combining the



**Figure 5.8 (b): Graphical solutions for implementation of media campaign against bandits, rehabilitation of bandits and use of military force reduce rehabilitated bandits**



**Figure 5.8 (c): Graphical solutions for implementation of media campaign against bandits, rehabilitation of bandits and use of military force to lessen the detained bandits**



**Figure 5.8 (d): The control profile of the control measures**

previously indicated control methods, the study enhanced the work of [9] and (Shuai, & Van Den, 2013). The positivity and uniqueness of the model equations' solutions were established, and it was found that they do indeed exist and are distinct.

The model's menace free equilibrium state were identified. The Banditry menace free equilibrium states was all found to be

$$E_B^0 = (S^0, B^0, D_T^{B^0}, R_E^{B^0}, M_F^0) = \left( \frac{\Lambda_1}{\mu}, 0, 0, 0, \frac{\Lambda_2}{\mu + \eta} \right)$$

By rigorously examining these, we get to the conclusion that the susceptible population and military forces alter accordingly with their recruitment rate to their mortality rate when there is no Banditry.

The basic reproduction number of the Banditry model was also analyzed and obtained to be

$$R_{0B} = \frac{\rho_3 \Lambda_1}{\Lambda(\mu + \sigma_1)}$$

As an interpretation, the basic reproduction number is given by the product of the rate of movement ( $\rho_3$ ) of the susceptible population to the Banditry compartment, the ratio of the recruitment rate ( $\Lambda_1$ ) into susceptible population to the sum of recruitment into susceptible population and into military compartment  $\left( \frac{\Lambda_1}{\Lambda} \right)$  and the average duration in Banditry class  $\left( \frac{1}{\mu + \sigma_1} \right)$ . It shows the

average number of people who can be initiated into Banditry by single Bandit member. If the population of Bandits  $B=0$  and  $R_{0B} < 1$ , then the population of Bandits disappear over time. And if

population of Bandits  $B=0$  and  $R_{0B} > 1$ , then the Bandits population persists.

In order to examine and analyze the ideal level for regulating the widespread media campaign against banditry, the rehabilitation of captured bandits, and the employment of military forces as control variables, we utilized the optimal control approach employing Pontryagin's Maximum Principle. The findings indicate that combining all three tactics is the most effective way to stop the banditry threat.

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