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Optimization Problem for Deteriorating Items Model with Stock Dependent Demand and Controllable Deterioration Under Two Warehouse Storage Facility

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ABSTRACT

In any inventory and production management, deterioration of physical goods is one of the important factors put into consideration. Therefore, it is very important to control and maintain inventories for decaying items. We considered inventory problem for deteriorating items where demand is stock dependent and deterioration is controllable under two warehouse storage facility, one of the warehouses is owned and the other one is rented. Goods are displayed in both warehouses but demands are met from the rented warehouse until the level of goods get to zero then demands are met from owned warehouse. We derived the optimal inventory management policies for the cases where; the demand is constant function, there is no investment in technology and there is no investment in technology and demand is also constant.

Keywords: Deteriorating items; stock; warehouse; demand; controllable deterioration.

1.0 INTRODUCTION

A lot of physical goods undergo decay or deterioration over time. For instance, commodities such as fruits, vegetables, food items etc., while kept in store undergo depletion by direct spoilage. Also, electronic goods, grains, chemicals, pharmaceutical products etc., undergo deterioration through gradual loss of potential or utility as time passes. Therefore, goods undergoing deterioration with passage of time can be found in real life situation, and in any inventory and production management, deterioration of physical goods is one of the important factors put into consideration. Hence, it is very important to control and maintain inventories for decaying items.

A set of mathematical models that describes the properties of a wide variety of inventory systems and as well as different methodologies studies that seek and analyze the best strategies that can be employed in inventories management is referred to as inventory theory (Luis *et al*., 2021). In this regard, Luis *et al.* (2021) studied an inventory model where demand rate put into consideration the effects of time and selling price. They assumed that the demand rate consists of two power functions. One of them depend on the selling price and the other depend on the time elapsed since the last inventory replenishment. The also assumed that shortages are allowed and fully backlogged, and determine the optimal inventory strategies by maximizing the profit function. They also presented numerical examples to verify their results.

In inventory management in practice, it might be more advantageous for the firm that customers have to wait for certain period until the next replenishment to get their order, so, here shortages are allowed and can be completely backlogged or partially backlogged i.e., any customer that come for transaction during the stock – out period is willing to wait until the next replenishment or some of them that come for transaction during the stock – out period are willing to wait until the next replenishment. Some of the work that have considered the concept of shortages and backlogging can be found in (San – Jose *et al*., 2020; San – Jose *et al*., 2019). For instance, Rajan and Uthayakumar (2017) considered an economic order quantity model which investigate the optimal pricing and replenishment policies for instantaneous deteriorating items with backlogging and trade credit under inflation. Here, they discussed profit maximization problem, and determined the optimal selling price, optimal order quantity and optimal replenishment time. Also, how learning effect affect rework policy, inflation and the time value of money has been considered by Amalesh *et al*. (2021) in their work titled "An imperfect production model for breakable multi – items with dynamic demand and learning effect on rework over random planning horizon". From their analysis, they concluded that manufacturers should adapt rework policy to avoid loss due to presence of defective items, decrease the total cost of the manufacturing system and increase the rework rate of imperfect items.

Banu *et al*. (2021) considered a supply chain model with stock dependent demand rate and trade credit financing. Meanwhile, it can also be observed that lowering the selling price of a commodity increases the customers demand for that commodity. The work of Halim *et al.* (2021) tried to explore this idea. Their work considered stock dependent demand and nonlinear price dependent demand for production inventory model for deteriorating items. The work of Shaikh *et al.* (2020) also considered production model for deteriorating item under trade credit policy. In their work, they investigated the effect of inflation and price dependent demand on the model and in the study carry out by Khanna *et al*. (2020), they derived the optimal preservation strategies in an inventory model with stock dependent demand and time dependent holding cost.

Pervin *et al.* (2019) developed an inventory models for deteriorating multi – items with stock dependent demand and price dependent demand under trade credit policy. So, lot of inventory models are developed with the assumption that deteriorating rate is either constant or time dependent. However, controllable deterioration is also an important strategy that can influence deterioration. Thus, studies that considered inventory model with controllable deterioration have received wide attention. In our work, we considered inventory problem for deteriorating items where demand is stock dependent and deterioration is controllable under two warehouse storage facility.

2.0 MODELS FORMULATION

2.1 Constants and Variables

S The ordering cost per order
c the purchasing cost per unit item
p the selling price per unit item
h_o The holding cost per unit time in Owned Warehouse (OW)
h_r The holding cost per unit time in Rented Warehouse (RW)
μ the deteriorating rate of the inventory in the rented warehouse
W The storage capacity of OW
$I_o(t)$ inventory level at time t in OW
$I_r(t)$ inventory level at time t in RW
t_1 The time at which the inventory in RW reaches level zero
T The length of replenishment cycle
q The replenishment quantity per replenishment
ω Maximum capital constraint
ξ Preservation technology cost per unit time for reducing deterioration rate in order to preserve the products ($0 \leq \xi \leq \omega$)
$X_1(t_1)$ The total profit function per unit time

2.2 Assumptions

i. Replenishment is instantaneous
ii. The time horizon of the inventory system is infinite
iii. There is no repair or replacement of deteriorated items during the period under consideration
iv. It is assumed that the items in the OW follow reduced deterioration rate $M(\xi)$ which is an increasing function of the preservation technology cost ξ where
v. $\lim_{\xi \to \infty} M(\xi) = \theta,$
vi. $M(\xi) = \theta(1 - e^{-a\xi})\ a > 0,$
vii. $\xi > 0$
viii. The demand rate function $D(t)$ is assumed to be dependent on the current inventory level, $D(t)$ is given by
ix. $D(t) = \alpha + \beta I(t),\ 0 \leq t \leq t_1$
x. $\alpha > 0$ and $0 < \beta < 1$ are termed scale and shape parameters respectively
xi. Shortages are not allowed.
xii. Due to different preservation environment, holding costs per unit item in RW and OW are different as a result, it is assumed that the holding cost per unit item in RW is lower than that of OW
xiii. Items are not transferred from RW to OW, instead demands are fulfilled directly from the RW until the inventory level in the RW reaches zero at time t_1 and after that, demands are fulfilled from OW up to time T at which inventory in OW reaches zero.

3.0 THE TWO – WAREHOUSE MODELS

Here, retailer can order q units in each replenishment cycle. Out of the quantity q, W units are stored in OW and the remaining quantity $q - W$ units are stored in RW. It is assumed that the retailer can also display the stock in OW to attract customers but demands are fulfilled from RW until the inventory in RW reaches zero at time t_1. Then, demands can now be fulfilled from OW until inventory level in OW reaches zero at time T.

Case I: Time period $0 \leq t \leq t_1$

In this period, the demands are fulfilled from RW. Therefore, the inventory level in the owned warehouse decreases due to deterioration only. Thus, the differential equation governing the model is as follows:

$$\frac{dI_O(t)}{dt} = -(\theta - m(\xi))I_O(t) \qquad (1)$$

$0 \leq t \leq t_1$

$I_O(0) = W$

Solving equation (1), we have

$$\frac{dI_O(t)}{(\theta - m(\xi))I_O(t)} = -dt$$

$$\int_0^t \frac{dI_O(t)}{(\theta - m(\xi))I_O(t)} = -\int_0^t dt = -t$$

$$\frac{1}{\theta - m(\xi)} \int_0^t \frac{du}{u} = -t$$

$$\int_0^t \frac{du}{u} = -(\theta - m(\xi))t$$

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$$\ln\left[\frac{\big(\theta - m(\xi)\big)I_O(t)}{\big(\theta - m(\xi)\big)I_O(0)}\right] = -\big(\theta - m(\xi)\big)t$$

$$I_O(t) = I_O(0)e^{-(\theta - m(\xi))t}$$

$$I_O(t) = We^{-(\theta - m(\xi))t} \qquad (2)$$

Also, in this time the inventory level in RW depletion is due to the combined effect of the demands and deterioration. Therefore, the differential equation governing the model is given by:

$$\frac{dI_r(t)}{dt} = -\mu I_r(t) - \big(\alpha + \beta I_r(t)\big) \qquad (3)$$

$$0 \le t \le t_1$$

$$I_r(t_1) = 0$$

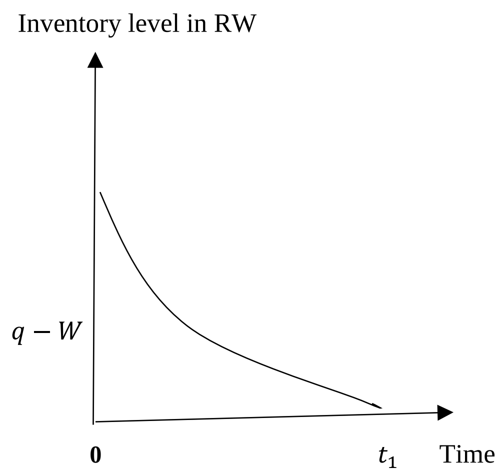

Figure 1. Inventory level for the rented warehouse

$$\frac{dI_r(t)}{dt} = -\mu I_r(t) - \big(\alpha + \beta I_r(t)\big)$$

$$= -\mu I_r(t) - \alpha - \beta I_r(t)$$

$$= -[\alpha + (\mu + \beta)I_r(t)]$$

$$\frac{dI_r(t)}{[\alpha + (\mu + \beta)I_r(t)]} = -dt$$

$$\int_t^{t_1} \frac{dI_r(t)}{[\alpha + (\mu + \beta)I_r(t)]} = -(t_1 - t)$$

$$\ln\left[\frac{\alpha + (\mu + \beta)I_r(t_1)}{\alpha + (\mu + \beta)I_r(t)}\right] = -(\mu + \beta)(t_1 - t)$$

$$\frac{\alpha}{\alpha + (\mu + \beta)I_r(t)} = e^{-(\mu + \beta)(t_1 - t)}$$

$$\alpha + (\mu + \beta)I_r(t) = \alpha e^{(\mu + \beta)(t_1 - t)}$$

$$(\mu + \beta)I_r(t) = \alpha(e^{(\mu + \beta)(t_1 - t)} - 1)$$

$$I_r(t) = \frac{\alpha(e^{(\mu + \beta)(t_1 - t)} - 1)}{\mu + \beta} \qquad (4)$$

Case II: The Period $t_1 \le t \le T$

In this period, owned warehouse inventory depletion is due to combined effect of the demand and deterioration. Hence, the governing differential equation is given by:

$$\frac{dI_o(t)}{dt} = -\big(\alpha + \beta I_o(t)\big)$$

$$-\big(\theta - m(\xi)\big)I_o(t) \qquad (5)$$

$$t_1 \le t \le T$$

$$I_o(T) = 0$$

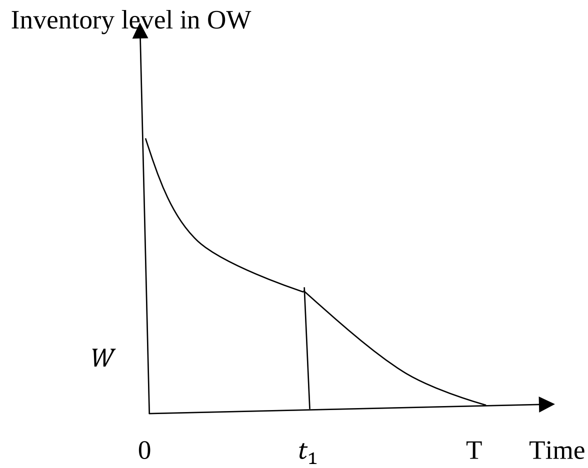

Figure 2. Inventory level for the owned warehouse

Solving Eq. (5), we have

$$\frac{dI_o(t)}{dt} = -\big(\alpha + \beta I_o(t)\big) - \big(\theta - m(\xi)\big)I_o(t)$$

$$= -\big(\alpha + [\beta + \theta - m(\xi)]I_o(t)\big)$$

$$\frac{dI_o(t)}{\big(\alpha + [\beta + \theta - m(\xi)]I_o(t)\big)} = -dt$$

$$\int_t^T \frac{dI_o(t)}{\big(\alpha + [\beta + \theta - m(\xi)]I_o(t)\big)} = -(T - t)$$

$$\ln\left[\frac{\alpha + [\beta + \theta - m(\xi)]I_o(T)}{\alpha + [\beta + \theta - m(\xi)]I_o(t)}\right]$$

$$= -[\beta + \theta - m(\xi)](T - t)$$

$$\frac{\alpha}{\alpha + [\beta + \theta - m(\xi)]I_o(t)} = e^{-[\beta + \theta - m(\xi)](T - t)}$$

$$I_o(t) = \frac{\alpha\big(e^{[\beta + \theta - m(\xi)](T - t)} - 1\big)}{\beta + \theta - m(\xi)} \qquad (6)$$

Note that from equation (2) and (6) we have that

$$I_o(t_1) = We^{-(\theta - m(\xi))t_1}$$

$$I_o(t_1) = \frac{\alpha\big(e^{[\beta + \theta - m(\xi)](T - t_1)} - 1\big)}{\beta + \theta - m(\xi)}$$

$$W[\beta + \theta - m(\xi)]e^{-(\theta - m(\xi))t_1}$$

$$= \alpha e^{[\beta + \theta - m(\xi)](T - t_1)} - \alpha$$

$$\frac{W[\beta + \theta - m(\xi)]e^{-(\theta - m(\xi))t_1} + \alpha}{\alpha}$$

$$= e^{[\beta + \theta - m(\xi)](T - t_1)}$$

$$\frac{W[\beta + \theta - m(\xi)]e^{\beta t_1} + \alpha e^{[\beta + \theta - m(\xi)]t_1}}{\alpha}$$

$$= e^{[\beta + \theta - m(\xi)]T}$$

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$$\ln\left[\frac{W[\beta+\theta-m(\xi)]e^{\beta t_1}+\alpha e^{[\beta+\theta-m(\xi)]t_1}}{\alpha}\right] = [\beta+\theta-m(\xi)]T$$

$$T = \frac{1}{\beta+\theta-m(\xi)} .\ln\left[\frac{W[\beta+\theta-m(\xi)]e^{\beta t_1}+\alpha e^{[\beta+\theta-m(\xi)]t_1}}{\alpha}\right]$$

3.1 Profit and Cost Components

Holding cost of the items in RW:

$$\text{Holding cost} = h_r\int_0^{t_1} I_r(t)dt$$
$$= h_r\int_0^{t_1}\left(\frac{\alpha(e^{(\mu+\beta)(t_1-t)}-1)}{\mu+\beta}\right)dt$$
$$= \frac{h_r\alpha}{\mu+\beta}\int_0^{t_1}\left(e^{(\mu+\beta)(t_1-t)}-1\right)dt$$
$$= \frac{h_r\alpha}{\mu+\beta}\left|\frac{e^{(\mu+\beta)(t_1-t)}}{-(\mu+\beta)}-t\right|_0^{t_1}$$
$$= \frac{h_r\alpha}{\mu+\beta}\left(\frac{e^{(\mu+\beta)t_1}-1}{\mu+\beta}-t_1\right) \quad (7)$$

Holding cost of the items in owned warehouse

$$\text{Holding cost} = h_o\int_0^{t_1} I_o(t)dt + h_o\int_{t_1}^{T} I_o(t)dt$$
$$= h_oW\int_0^{t_1} e^{-(\theta-m(\xi))t}\,dt + \frac{h_o\alpha}{\beta+(\theta-m(\xi))}\times\int_{t_1}^{T}\left(e^{[\beta+\theta-m(\xi)](T-t)}-1\right)dt$$
$$= h_0W\left|-\frac{e^{-(\theta-m(\xi))t}}{\theta-m(\xi)}\right|_0^{t_1} + \frac{h_o\alpha}{\beta+\theta-m(\xi)}\left|-\frac{e^{[\beta+\theta-m(\xi)](T-t)}}{\beta+\theta-m(\xi)} - t\right|_{t_1}^{T}$$
$$= h_0W\left[\left(-\frac{e^{-(\theta-m(\xi))t_1}}{\theta-m(\xi)}\right)-\left(-\frac{1}{\theta-m(\xi)}\right)\right] + \frac{h_o\alpha}{\beta+\theta-m(\xi)}\left[\left(-\frac{1}{\beta+\theta-m(\xi)}-T\right) - \left(-\frac{e^{[\beta+\theta-m(\xi)](T-t_1)}}{\beta+\theta-m(\xi)}-t_1\right)\right]$$
$$= \frac{h_0W}{\theta-m(\xi)}\left[1-e^{-(\theta-m(\xi))t_1}\right] + \frac{h_o\alpha}{\beta+\theta-m(\xi)}\times\left[\frac{\left(e^{[\beta+\theta-m(\xi)](T-t_1)}-1\right)}{\beta+\theta-m(\xi)}+t_1-T\right] \quad (8)$$

$$\text{Ordering cost} = S \quad (9)$$
$$\text{Purchasing cost} = cQ$$
$$Q = I_r(0)+I_o(0) = \frac{\alpha(e^{(\mu+\beta)(t_1-0)}-1)}{\mu+\beta}+We^{-(\theta-m(\xi))\cdot 0} = \frac{\alpha(e^{(\mu+\beta)t_1}-1)}{\mu+\beta}+W$$
$$\text{Purchasing cost} = cQ = \frac{c\alpha(e^{(\mu+\beta)t_1}-1)}{\mu+\beta}+cW \quad (10)$$

$$\text{Sales revenue} = p\int_0^{T} D(t)\,dt = p\left[\int_0^{t_1} D(t)\,dt + \int_{t_1}^{T} D(t)\,dt\right]$$
$$= p\int_0^{t_1}\left(\alpha+\beta I_r(t)\right)dt + p\int_{t_1}^{T}\left(\alpha+\beta I_o(t)\right)dt$$
$$= p\int_0^{t_1}\left(\alpha+\frac{\alpha\beta}{\mu+\beta}\left(e^{(\mu+\beta)(t_1-t)}-1\right)\right)dt + p\int_{t_1}^{T}\left(\alpha + \frac{\alpha\beta}{\beta+\theta-m(\xi)}\left(e^{[\beta+\theta-m(\xi)](T-t)}-1\right)\right)dt$$
$$= p\left|\alpha t+\frac{\alpha\beta}{\mu+\beta}\left(-\frac{e^{(\mu+\beta)(t_1-t)}}{\mu+\beta}\right)-\frac{\alpha\beta t}{\mu+\beta}\right|_0^{t_1} + p\left|\alpha t+\frac{\alpha\beta}{\beta+\theta-m(\xi)}\times\left(-\frac{e^{[\beta+\theta-m(\xi)](T-t)}}{\beta+\theta-m(\xi)}\right)-\frac{\alpha\beta t}{\beta+\theta-m(\xi)}\right|_{t_1}^{T}$$
$$= \frac{\alpha\beta p}{[\mu+\beta]^2}\left(e^{(\mu+\beta)t_1}-1\right) + \frac{\alpha p(\theta-m(\xi))T}{\beta+\theta-m(\xi)} + \frac{\alpha\beta p\left(e^{[\beta+\theta-m(\xi)](T-t_1)}-1\right)}{[\beta+\theta-m(\xi)]^2} + \frac{\alpha\beta p t_1}{\beta+\theta-m(\xi)} - \frac{\alpha\beta p t_1}{\mu+\beta} \quad (11)$$

Now, the annual profit function can be expressed as:

$$X_1(t_1) = \frac{1}{T}(\text{sales revenue} - \text{purchasing cost} - \text{ordering cost} - \text{stock holding cost in RW}$$

−stock holding cost in OW)

$$=\frac{1}{T}\left\{\frac{\alpha\beta p}{[\mu+\beta]^2}\left(e^{(\mu+\beta)t_1}-1\right)+\frac{\alpha p(\theta-m(\xi))T}{\beta+\theta-m(\xi)}+\frac{\alpha\beta p\left(e^{[\beta+\theta-m(\xi)](T-t_1)}-1\right)}{[\beta+\theta-m(\xi)]^2}+\frac{\alpha\beta p t_1}{\beta+\theta-m(\xi)}-\frac{\alpha\beta p t_1}{\mu+\beta}-\left(\frac{c\alpha\left(e^{(\mu+\beta)t_1}-1\right)}{\mu+\beta}+cW\right)-S-\frac{h_r\alpha}{\mu+\beta}\left(\frac{e^{(\mu+\beta)t_1}-1}{\mu+\beta}-t_1\right)-\left(\frac{h_0W}{\theta-m(\xi)}\left[1-e^{-(\theta-m(\xi))t_1}\right]+\frac{h_o\alpha}{\beta+\theta-m(\xi)}\left[\frac{\left(e^{[\beta+\theta-m(\xi)](T-t_1)}-1\right)}{\beta+\theta-m(\xi)}+t_1-T\right]\right)\right\} \quad (12)$$

4.0 PROFIT OPTIMIZATION

Total average profit will be maximum (the necessary condition) if

$$\frac{d}{dt_1}X_1(t_1)=0 \quad (13)$$

Provided that the value of t_1 satisfies the condition (the sufficient condition)

$$\left[\frac{d^2}{dt_1^2}X_1(t_1)\right]_{t_1=t_1^*}<0 \quad (14)$$

Our aim here is to determine t_1 which maximizes the total average profit $X_1(t_1)$. Thus, on solving Equation (13), we find the optimum value(s) of t_1 for which the total variable inventory profit is maximum.

Theorem 4.1

If the holding cost per unit time in Owned Warehouse (OW), the holding cost per unit time in Rented Warehouse (RW) and the deteriorating rate in OW $m(\xi)$ satisfy

i. $h_O>\beta p$
ii. $h_r>\beta p-c(\mu+\beta)$
iii. $m(\xi)=\theta$

Then profit is maximized.

Proof:

We have that

$$\frac{\partial}{\partial t_1}X_1(t_1)=\frac{1}{T}\left\{\left(\frac{\alpha\beta p}{[\mu+\beta]^2}\cdot(\mu+\beta)e^{(\mu+\beta)t_1}\right)+\left(\frac{\alpha\beta p e^{[\beta+\theta-m(\xi)](T-t_1)}}{[\beta+\theta-m(\xi)]^2}\cdot-(\beta+\theta-m(\xi))\right)+\frac{\alpha\beta p}{\beta+\theta-m(\xi)}-\frac{\alpha\beta p}{\mu+\beta}-\left(\frac{c\alpha e^{(\mu+\beta)t_1}}{\mu+\beta}\cdot(\mu+\beta)\right)-\frac{h_r\alpha}{\mu+\beta}\left(\left[(\mu+\beta)\cdot\frac{e^{(\mu+\beta)t_1}}{\mu+\beta}\right]-1\right)-\frac{h_0W[\theta-m(\xi)]}{\theta-m(\xi)}\left(e^{-(\theta-m(\xi))t_1}\right)-\left(\frac{h_o\alpha}{\beta+\theta-m(\xi)}\left[\left(-[\beta+\theta-m(\xi)]\cdot\frac{e^{[\beta+(\theta-m(\xi))](T-t_1)}}{\beta+(\theta-m(\xi))}\right)+1\right]\right)\right\}$$

$$=\frac{1}{T}\left\{\frac{\alpha\beta p e^{(\mu+\beta)t_1}}{\mu+\beta}-\frac{\alpha\beta p e^{[\beta+\theta-m(\xi)](T-t_1)}}{\beta+\theta-m(\xi)}+\frac{\alpha\beta p}{\beta+\theta-m(\xi)}-\frac{\alpha\beta p}{\mu+\beta}-c\alpha e^{(\mu+\beta)t_1}-\frac{h_r\alpha}{\mu+\beta}\left(e^{(\mu+\beta)t_1}-1\right)-h_0W\left[e^{-(\theta-m(\xi))t_1}\right]-\frac{h_o\alpha}{\beta+\theta-m(\xi)}\left[-e^{[\beta+(\theta-m(\xi))](T-t_1)}+1\right]\right\}$$

$$=\frac{1}{T}\left\{\frac{\alpha\beta p e^{(\mu+\beta)t_1}}{\mu+\beta}-\frac{\alpha\beta p e^{[\beta+\theta-m(\xi)](T-t_1)}}{\beta+\theta-m(\xi)}+\frac{\alpha\beta p}{\beta+\theta-m(\xi)}-\frac{\alpha\beta p}{\mu+\beta}-c\alpha e^{(\mu+\beta)t_1}-\frac{h_r\alpha e^{(\mu+\beta)t_1}}{\mu+\beta}+\frac{h_r\alpha}{\mu+\beta}-h_0We^{-(\theta-m(\xi))t_1}+\frac{h_o\alpha e^{[\beta+(\theta-m(\xi))](T-t_1)}}{\beta+\theta-m(\xi)}-\frac{h_o\alpha}{\beta+\theta-m(\xi)}\right\}=0 \quad (15)$$

Therefore,

$$\frac{d^2}{dt_1^2}X_1(t_1)=\frac{1}{T}\left\{\alpha\beta p e^{(\mu+\beta)t_1}+\alpha\beta p e^{[\beta+\theta-m(\xi)](T-t_1)}-c\alpha(\mu+\beta)e^{(\mu+\beta)t_1}-h_r\alpha e^{(\mu+\beta)t_1}+(\theta-m(\xi))h_0We^{-(\theta-m(\xi))t_1}-h_o\alpha e^{[\beta+\theta-m(\xi)](T-t_1)}\right\} \quad (16)$$

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$$= \frac{1}{T}\{(\beta p - c(\mu+\beta) - h_r)\alpha e^{(\mu+\beta)t_1} + (\beta p - h_o)\alpha e^{[\beta+\theta-m(\xi)](T-t_1)}\} + (\theta - m(\xi))h_0 W e^{-(\theta-m(\xi))t_1}\} < 0 \quad (17)$$

For

i. $h_O > \beta p$
ii. $h_r > \beta p - c(\mu+\beta)$
iii. $m(\xi) = \theta$

Equation (17) satisfies the sufficient condition that maximizes the objective function.

Theorem 4.2

If the demand is a constant function and

$$(\theta - m(\xi))h_0 W e^{-(\theta-m(\xi))t_1} < \alpha[(c\mu + h_r)e^{\mu t_1} + h_o e^{[\theta-m(\xi)](T-t_1)}]$$

Then

$$\frac{d^2}{dt_1^2}X_1(t_1) < 0$$

Proof:

From Equation (16), we have

$$\frac{d^2}{dt_1^2}X_1(t_1) = \frac{1}{T}\{\alpha\beta p e^{(\mu+\beta)t_1} + \alpha\beta p e^{[\beta+\theta-m(\xi)](T-t_1)} - c\alpha(\mu+\beta)e^{(\mu+\beta)t_1} - h_r\alpha e^{(\mu+\beta)t_1} + (\theta - m(\xi))h_0 W e^{-(\theta-m(\xi))t_1} - h_o\alpha e^{[\beta+\theta-m(\xi)](T-t_1)}\}$$

Therefore, for $D = \alpha$, we have

$$\frac{d^2}{dt_1^2}X_1(t_1) = \frac{1}{T}\{0 + 0 - c\alpha(\mu+0)e^{\mu t_1} - h_r\alpha e^{\mu t_1} + (\theta - m(\xi))h_0 W e^{-(\theta-m(\xi))t_1} - h_o\alpha e^{[\theta-m(\xi)](T-t_1)}\}$$

$$= \frac{1}{T}\{-c\alpha\mu e^{\mu t_1} - h_r\alpha e^{\mu t_1} + (\theta - m(\xi))h_0 W e^{-(\theta-m(\xi))t_1} - h_o\alpha e^{[\theta-m(\xi)](T-t_1)}\}$$

$$= \frac{1}{T}\{(\theta - m(\xi))h_0 W e^{-(\theta-m(\xi))t_1} - c\alpha\mu e^{\mu t_1} - h_r\alpha e^{\mu t_1} - h_o\alpha e^{[\theta-m(\xi)](T-t_1)}\}$$

$$= \frac{1}{T}\{(\theta - m(\xi))h_0 W e^{-(\theta-m(\xi))t_1} - \alpha[(c\mu + h_r)e^{\mu t_1} + h_o e^{[\theta-m(\xi)](T-t_1)}]\} < 0$$

For

$$(\theta - m(\xi))h_0 W e^{-(\theta-m(\xi))t_1} < \alpha[(c\mu + h_r)e^{\mu t_1} + h_o e^{[\theta-m(\xi)](T-t_1)}]$$

Theorem 4.3

If there is no investment in technology and

i. $h_O > \beta p$
ii. $h_r > \beta p - c(\mu+\beta)$

Then

$$\frac{d^2}{dt_1^2}X_1(t_1) < 0$$

proof:

from Equation (16), we have

$$\frac{d^2}{dt_1^2}X_1(t_1) = \frac{1}{T}\{\alpha\beta p e^{(\mu+\beta)t_1} + \alpha\beta p e^{[\beta+\theta-m(\xi)](T-t_1)} - c\alpha(\mu+\beta)e^{(\mu+\beta)t_1} - h_r\alpha e^{(\mu+\beta)t_1} + (\theta - m(\xi))h_0 W e^{-(\theta-m(\xi))t_1} - h_o\alpha e^{[\beta+\theta-m(\xi)](T-t_1)}\}$$

$$= \frac{1}{T}\{\alpha\beta p e^{(\mu+\beta)t_1} + \alpha\beta p e^{[\beta](T-t_1)} - c\alpha(\mu+\beta)e^{(\mu+\beta)t_1} - h_r\alpha e^{(\mu+\beta)t_1} + (0)h_0 W e^{-(\theta-0)t_1} - h_o\alpha e^{[\beta](T-t_1)}\}$$

$$= \frac{1}{T}\{\alpha\beta p e^{(\mu+\beta)t_1} + \alpha\beta p e^{\beta(T-t_1)} - c\alpha(\mu+\beta)e^{(\mu+\beta)t_1} - h_r\alpha e^{(\mu+\beta)t_1} + 0 - h_o\alpha e^{\beta(T-t_1)}\}$$

$$= \frac{1}{T}\{(\beta p - c(\mu+\beta) - h_r)\alpha e^{(\mu+\beta)t_1} + (\beta p - h_o)\alpha e^{\beta(T-t_1)}\} < 0$$

For

i. $h_O > \beta p$
ii. $h_r > \beta p - c(\mu+\beta)$

Theorem 4.4

If there is no investment in technology and demand is constant, and

i. $h_O > \beta p$
ii. $h_r > \beta p - c(\mu+\beta)$

Then

$$\frac{d^2}{dt_1^2}X_1(t_1) < 0$$

proof:

from Equation (16), we have

$$\frac{d^2}{dt_1^2}X_1(t_1) = \frac{1}{T}\{\alpha\beta p e^{(\mu+\beta)t_1} + \alpha\beta p e^{[\beta+\theta-m(\xi)](T-t_1)} - c\alpha(\mu+\beta)e^{(\mu+\beta)t_1} - h_r\alpha e^{(\mu+\beta)t_1} + (\theta - m(\xi))h_0 W e^{-(\theta-m(\xi))t_1} - h_o\alpha e^{[\beta+\theta-m(\xi)](T-t_1)}\}$$

From $D = \alpha$ and $\xi = 0$, we have

$$\frac{d^2}{dt_1^2}X_1(t_1) = \frac{1}{T}\{0 + 0 - c\alpha(\mu+0)e^{(\mu+0)t_1}$$

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$$-h_r\alpha e^{(\mu+0)t_1}$$
$$+(0)h_0 W e^{-(0)t_1}$$
$$-h_o\alpha e^{[0](T-t_1)}\}$$
$$=\frac{1}{T}\{-c\alpha\mu e^{\mu t_1}-h_r\alpha e^{\mu t_1}$$
$$-h_o\alpha\}<0$$

5.0 CONCLUSION

In this paper, we have derived the optimal inventory policies for deteriorating items with stock dependent demand and controllable deterioration under two warehouse storage facility that maximizes the objective function of the deteriorating items model with stock dependent demand and controllable deterioration under two warehouse storage facility. The owned warehouse have some facilities that control deterioration while such condition is not available in the rented warehouse. We obtained the optimal inventory management policy that maximizes the objective function (average total profit function) under the prescribe conditions.

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