

# Impacts of Casson Fluid Flow and Heat Source/Sink in an Oscillating Plate with Unequal Wall Temperature

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Received on: August, 2024 Revised and Accepted on: September, 2024

Published on: October, 2024

## ABSTRACT

Casson fluid is commonly used in many notable technological and industrial properties, such as synthetic lubricants, specific oil paints, biological fluids, diverse polymer solutions to mention few. The Casson fluid is considered to be one of the most prominent types of fluids within the category of non-Newtonian substances. The impact of Casson fluid impact on hydromagnetic oscillatory flow along a permeable plate immersed in porous medium is investigated in the optically thin thermal radiation regime. The solutions of the dimensionless equations have been obtained. In view of the assumed oscillatory pressure gradient, the resultant linear partial differential equations were reduced to a boundary-valued-problem where the unsteady flow is superimposed on the mean steady flow. The influence of controlling parameters dictating the flow behaviour have been demonstrated graphically and explained thoroughly. It is revealed from the computational analysis that the function of Casson fluid parameter is to diminish the fluid velocity. Additionally, the skin friction is increased at both walls as the suction/injection parameter is increased. Interestingly, the results obtained for limiting case in this research is consistent with previous literature, thereby establishing the accuracy and validity of the current investigation.

**Keywords:** Casson parameter, heat source/sink, oscillatory flow, Darcy porous medium, Slip parameter

## 1.0 INTRODUCTION

The term "Casson fluid" explains a kind of non-Newtonian fluid characterized by variable viscosity. The dominance of viscous force in this system can be attributed to the action of variable viscosity in the fluid. Fluids commonly employed in many applications include paints, diverse polymer solutions, blood, honey, etc. Typical instances of Casson fluids encompass synthetic lubricants, mud extraction, clay coatings, and biomedical fluids. The Casson fluid simulation that are readily accessible are classified based on their distinct rheological properties, including Oldroyd-B, Eyring-Powell, Oldroyd-A, Maxwell, Carreau, Jeffrey and Burger. In the field of modern technology, there are some flow features that defy comprehension when analysed solely through the lens of the Newtonian flow model. Consequently, the utilization of the non-Newtonian fluid notion is more advantageous. Some noteworthy works like Amaraikannan et al. (2021a), shahzad et al. (2021), Amaraikannan et al. (2021b) and Ali et al. (2021a) provide valuable insights for readers on this topic. Additionally, Ojemer et al. (2023) studied MHD flow of an electrically conductive Casson fluid with thermal radiation effect in a vertical porous plate. Also, Nandeppanavar (2018) described the movement pattern of a Casson fluid through a moving plate. Sheikh et al. (2020) have presented a study on the implication of the flow of a non-Newtonian fluid across a moving sheet. Ahmad et al. (2019) introduced a novel

approach to modelling a Casson fluid with fractional derivatives in a more recent study. Further, Sarkar and Endalew (2019, 2020), Hamid et al. (2019), Das et al. (2021), Amjad et al. (2021), and Sarkar et al. (2020) have published the flow behavior of the Casson fluid in various physical settings.

Magneto-hydrodynamics (MHD) is the study of electrically conducting liquids, namely, salty water, electrolytes, plasma, and liquid metals. This kind of fluid has a variety of scientific and technological significance, namely, in production of crystals, reactor cooling, magnetic drug targeting and MHD sensors. The empirical analysis of contemporary MHD flow in a laboratory was first done by Hartmann and Lazarus (1937). This investigation established the basic ideas for the creation of several MHD equipment, such as MHD pumps, MHD generators, brakes, flow meters and so on. Buoyancy-induced flow along a magnetized oscillating system with heat transfer across different geometrical settings, has been a topic of growing interest among many researchers today owing to its vast applications in geophysics, solid mechanics, hydrology, oil recovery and in the field of engineering, to cite a few (Chitra and Suhasini, 2018). With these concerns in mind, Ahmed et al. (2024) recently outlined the influence of Jeffrey fluid flow with MHD oscillatory system affected by unequal wall temperature along a porous channel. Omokhuale et al. (2024) performed the analysis of viscous dissipation on a thermally and chemically controlled nanofluid through a boundary layer region in an oscillating

system while Usman *et al.* (2024) examined the impact of thermophoresis on magnetized oscillatory system of a buoyancy-induced flow with nanoparticles along a boundary layer regime. In a different paper, Usman and Sanusi (2023) investigated the impacts of non-Newtonian nanofluid flows across a semi-infinite flat plate entrenched in a porous media affected by heat radiation, Soret, and pressure terms. Sharma *et al.* (2022) discussed the numerical study of MHD oscillatory flow of viscous dissipative fluid along an upstanding channel entrenched with porous medium due to heat source and thermal radiation impacts. Falade *et al.* (2017) deliberated on suction/injection effects in a magnetized oscillatory slip flow along a vertical plate with unequal wall temperature confined to an applied magnetic field. Hamza *et al.* (2011) considered the influence of chemical reaction and slip condition on a transient MHD heat and mass transfer flow via a vertical plate filled with porous medium in an oscillating system.

The concept of thermal radiation effect, which is the electromagnetic wave radiation that a surface generates because of its heat, is gaining growing attention, especially when a magnetic field is applied, due to its relevance in constructing different advanced energy conversion systems capable of operating at high temperatures (Jamaludin *et al.* 2020). Other areas of possible advantages include nuclear plants, solar technology, spacecraft aerodynamics, to state a few. In view of this, several scholars have carried out investigations on the impact of heat radiation in a number of physical configurations. With the numerous advantages of thermal radiation in mind, Hamza *et al.* (2023) recently scrutinized the consequences of thermal radiation and super-hydrophobicity on a magnetized natural convection fluid through a heated porous superhydrophobic microchannel. Shah *et al.* (2023) investigated the impact of heat transfer in MHD Casson flow in the presence of thermal and chemical reactions influenced by thermal fluid properties. Using Darcy's model, Gireesha *et al.* (2020) outlined how thermal radiation and free convection affected the flow of a water-based hybrid nanofluid containing nanoparticles through a porous vertical channel, and Goud *et al.* (2023) examined the analysis of transient MHD flow through a permeable medium across an upright plate in the context of the coexistence of

viscous dissipation and thermal radiation effects. The impacts of thermal radiation, heat generation, and an induced magnetic field on the free convection of a couple stress fluid in a flux-isothermal upstanding plate have been analysed by Hasan *et al.* (2020) employing the method of indeterminate coefficient. In the presence of thermal radiation, Bejawada and Nandeppanavar (2022) studied the effects of the MHD heat transfer problem on the micropolar fluid through a vertically permeable moving plate. Parthiban and Pasad (2023) outlined a theoretical investigation of radiative-convection effects on MHD fluid flow in a heated square enclosure having a non-Darcy square cavity in the coexistence of the Hall effect and the heat source or sink. Using a spectral relaxation method, Haroun *et al.* (2017) highlighted the impact of heat radiation on magnetized mixed convection nanofluid flow along a moving plate.

The objective of this article is to modify the work of Falade *et al.* (2017) by incorporating the effects of Casson fluid and heat source/sink on MHD oscillatory flow equipped with suction/injection and porous medium factors. The outcomes of this type of research would be very relevant in engineering and industrial applications, particularly for exploration of crude oil from petroleum products, thermal insulation, gas turbines and drainage, fossil fuel combustion, to mention a few.

## 2.0 PROBLEM FORMULATION

Envisage the steady natural convection flow of a Casson fluid within an upstanding permeable channel in an oscillating system affected by an applied transverse magnetic field. It is thought of that there is no applied voltage which signifies the non-involvement of an electric field. The flow is assumed to be in the  $x$ -direction which is taken along the plate in the upward direction, the  $y$ -axis is perpendicular to it and the  $z$  axis along the wideness of the channel as sketched in Figure 1. Also, it is assumed that the whole system is rotating with a constant vector  $\Omega$  about  $y$ -axis. Since it is presumed that the plate surface is semi-infinite, the flow variables are functions of  $y$  only. Following Falade *et al.* (2017) and taking into account the Casson fluid parameter, while obeying the Boussinesq approximation, the resultant equations of this problem can be modelled as:

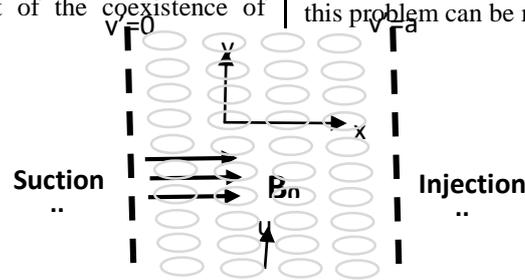


Figure 1. Physical configuration of the flow

$$\frac{\partial u'}{\partial t'} - v_0 \frac{\partial u'}{\partial y'} = -\frac{1}{\rho} \frac{dP}{dx'} + v \left(1 + \frac{1}{\xi}\right) \frac{\partial^2 u'}{\partial y'^2} - \frac{v}{K} u' - \frac{\sigma_e B_0^2 u'}{\rho} + g\beta(T' - T_0) \quad (1)$$

$$\frac{\partial T'}{\partial t'} - v_0 \frac{\partial T'}{\partial y'} = \frac{k_f}{\rho c_p} \frac{\partial^2 T'}{\partial y'^2} + \frac{4\alpha^2}{\rho c_p} (T' - T_0) - \frac{Q_0}{\rho c_p} (T' - T_0) \quad (2)$$

With the boundary condition  $T = T_1$

$$u' = \frac{\sqrt{K}}{\alpha_s} \frac{\partial u'}{\partial y'}, T = T_0 \text{ on } y' = 0 \quad (3)$$

$$u' = 0, T' = T_1 \text{ on } y' = a \quad (4)$$

While the dimensionless quantities used are provided below:

$$(x \ y) = \frac{(x', y')}{h}, u = \frac{hu'}{v}, t = \frac{vt'}{h^2}, p = \frac{h^2 p'}{\rho v^2}, Gr = \frac{g\beta(T_1 - T_0)h^3}{v^2}, Pr = \frac{h^2 p'}{\rho v^2}, \theta = \frac{T - T_0}{T_1 - T_0} \\ \delta = \frac{4x^2 h^2}{\rho c_p v}, \gamma = \frac{\sqrt{K}}{\alpha_s h}, Ha^2 = \frac{\sigma_e B_0^2 h^2}{\rho v}, Z = \frac{Q_0 h^2}{k}, Da = \frac{K}{h^2}, s = \frac{v_0 h}{v} \quad (5)$$

Inserting equation (5) into equations (1 – 4), we have the dimensionless governing equations as follows:

$$\frac{\partial u}{\partial t} - s \frac{\partial u}{\partial y} = -\frac{dP}{dx} + \left(1 + \frac{1}{\xi}\right) \frac{\partial^2 u}{\partial y^2} - \left(Ha^2 + \frac{1}{Da}\right) u + Gr\theta \quad (6)$$

$$\frac{\partial \theta}{\partial t} - s \frac{\partial \theta}{\partial y} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial y^2} + (D - Z)\theta \quad (7)$$

With the appropriate boundary conditions

$$\left. \begin{aligned} u &= \gamma \frac{du}{dy}, \quad \theta = 0 \text{ on } y = 0 \\ u &= 0, \quad \theta = 1 \text{ on } y = 1 \end{aligned} \right\} \quad (8)$$

### 3.0 SOLUTION PROCEDURE

We employed the theory of simultaneous differential equations to solve the resultant linear partial differential equations restricted to relevant boundary condition, after the unsteady flow is superimposed on the mean steady flow, so that in the neighbourhood of the plate, and assuming that an oscillatory pressure gradient as shown eqn (9), the solutions of velocity and temperature is in the form:

$$-\frac{dP}{dx} = \lambda e^{i\omega t}, \quad u(t, y) = u_0(y) e^{i\omega t}, \quad \theta(t, y) = \theta_0(y) e^{i\omega t} \quad (9)$$

where  $\lambda$  is any constant, and  $\omega$  is the frequency of oscillation (See Falade *et al.* (2017)). In view of (9), eqns (6 – 8) reduced to a boundary-valued-problem as follows:

$$u_0'' + \frac{s}{a_1} u_0' - \frac{1}{a_1} \left(Ha^2 + \frac{1}{Da} + i\omega\right) u_0 = -\frac{\lambda}{a_1} - \frac{Gr}{a_1} \theta \quad (10)$$

$$\theta_0'' + sPr\theta_0' - (D + Z - i\omega)\theta_0 = 0 \quad (11)$$

Subject to relevant boundary conditions:

$$\left. \begin{aligned} u_0 &= \gamma u_0', \quad \theta_0 = 0 \text{ on } y = 0 \\ u_0 &= 0, \quad \theta_0 = 1 \text{ on } y = 1 \end{aligned} \right\} \quad (12)$$

The exact solutions of temperature and velocity is obtained as:

$$\theta(t, y) = (J_1 e^{m_1 y} + J_2 e^{m_2 y}) e^{i\omega t} \quad (13)$$

$$u(t, y) = (L_1 e^{m_3 y} + L_2 e^{m_4 y} + L_3 + L_4 e^{m_1 y} + L_5 e^{m_2 y}) e^{i\omega t} \quad (14)$$

The heat transfer rate and frictional force is also computed as follows:

$$Nu = \frac{\partial \theta}{\partial y} = (J_1 m_1 e^{m_1 y} + J_2 m_2 e^{m_2 y}) e^{i\omega t} \quad (15)$$

$$S_f = \frac{\partial u}{\partial y} = (L_1 m_3 e^{m_3 y} + L_2 m_4 e^{m_4 y} + m_1 L_4 e^{m_1 y} + m_2 L_5 e^{m_2 y}) e^{i\omega t} \quad (16)$$

The constants used are indicated in the appendix section

#### 4.0 RESULTS AND DISCUSSION

The performance assessment of Casson fluid and heat source/sink on magnetized oscillating system equipped with radiation and porous medium impacts on natural convection flowing through an immeasurable upstanding permeable plate has been performed. The flow is instigated by buoyancy-induced growing pressure gradient along an upward facing plate. In order to point out the effects of physical parameters such as; Casson fluid parameter  $\xi$ , suction parameter  $s$ , magnetic parameter  $Ha$  and thermal Grashof number  $Gr$ , on the flow behaviours, computation of the flow fields is carried out. The influences of the major controlling parameters on the temperature and velocity distributions have been presented and discussed in Figures 4.2 to 4.12. The main default values selected for this analysis as they relate to real life applications are  $\xi = 0.1$ ,  $Z=0.5$ ,  $s=1$ ,  $Ha=1$ ,  $\delta = 1$ ,  $Da=1$ ,  $\omega = \pi$  and  $Gr=1$ . The graphical comparison of the work of Falade *et al.* (2017) and the present investigation is portrayed in Figure 4.1. The comparison displays an excellent agreement for the limiting case when Casson fluid and heat source/sink are parameters  $\xi = 1000$  and 0 respectively.

The action of Casson fluid parameter on the fluid velocity is depicted in Figure 4.2. It can clearly be seen that increasing Jeffrey fluid parameter, the velocity decreases. The velocity gradient for the application of various values of magnetic effect and is illustrated in Figure 4.3. The action of magnetic field perpendicular to the flow in an electrically conducting fluid produces a Lorentz force, which opposes the flow. With the aid of Figure 4.4, we comprehend the behaviour of fluid velocity as the Darcy porous medium is varied. It is apparent from this diagram that the fluid motion grows

as Darcy number is raised. This is true since, with stronger permeability of the porous material, the barriers placed on the flow path reduces, thereby encouraging free flow leading to stronger fluid speed. This action makes the fluid boundary wall and thickness to rise, which in turn escalates the fluid velocity. Thermal radiation's impact on the fluid temperature and velocity are seen in Figure 4.5 and Figure 4.6 respectively. It is noteworthy to report that when the thermal radiation is increased, the temperature of the fluid is notice to improve. This is attributable to the heat transport from the upper surface to the fluid because the fluid takes in its own radiations. Further, as displayed in Figure 4.6, the fluid motion is accelerating as a function of growing thermal radiation effect owing to heat production that elevates the flow movement. This is so because the heat emitted from the heated wall strengthens the fluid particles. The consequences of varying heat source/sink parameter are demonstrated in Figure 4.7 and Figure 4.8 respectively. From the sketch in Figure 4.7, it is viewed that mounting level of heat source/sink parameter encourages the fluid temperature. The concavity with the rise in the heat source/sink parameter is due to the direction of temperature flow from the hot plate towards the cold wall. Similarly, it is evident that increasing the heat source/sink raises the fluid velocity towards the cold wall as shown in Figure 4.8. Figure 4.9 explains that increasing the frequency of oscillation retards the fluid temperature inside the channel which is due to the weakening in the heat transfer amount as the heating frequency. The effect of Casson fluid parameter is demonstrated in Figure 4.10. It is seen that a rise in the skin friction is established in the cold plate while a reverse attribute happens in the heated wall. However, a point of intersection is viewed near the middle of the vertical channel.

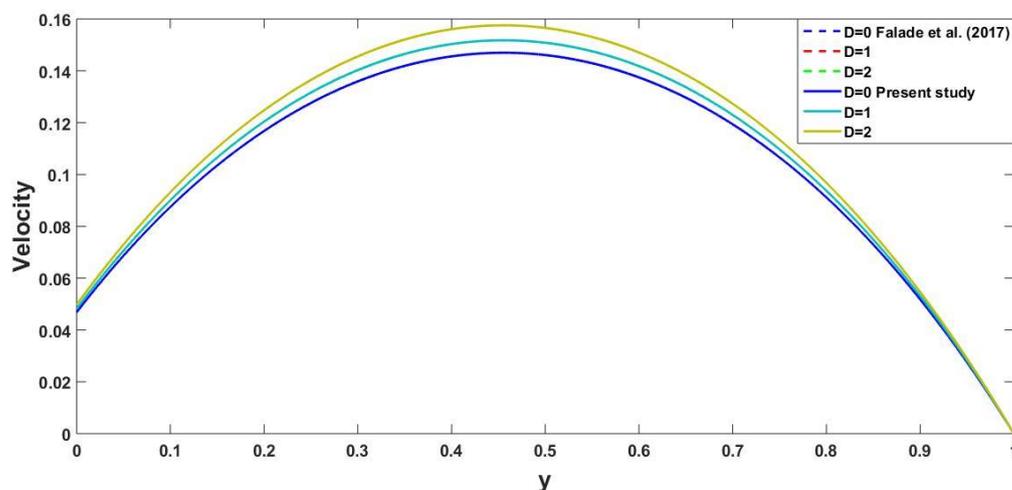


Figure 4.1: Comparison between the current analysis and that of

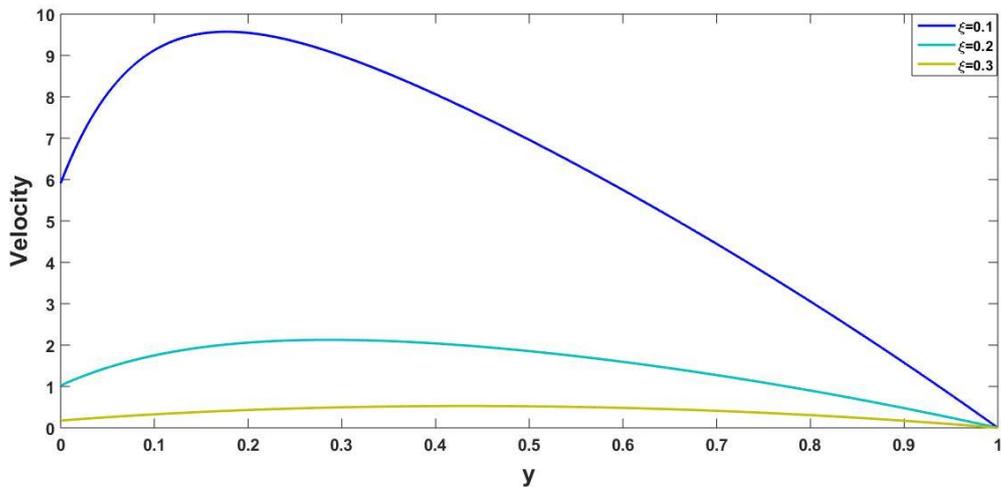


Figure 4.2: Variation of Casson parameter on Velocity profile

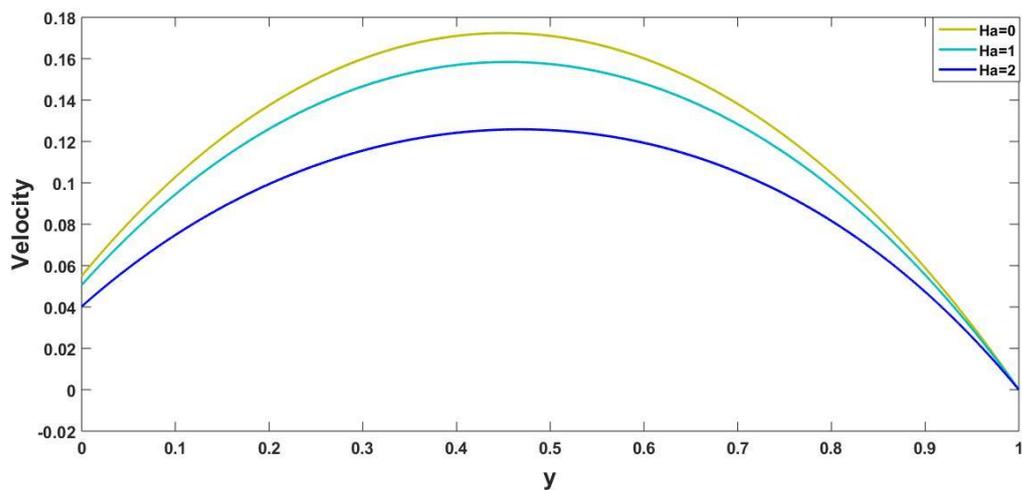


Figure 4.3: Variation of Hartmann number on Velocity profile

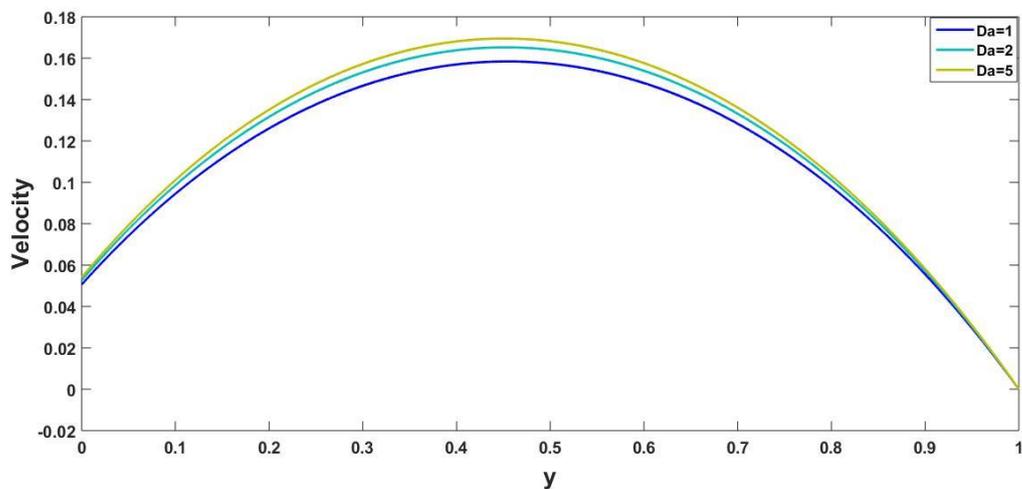


Figure 4.4: Variation of Darcy number on Velocity profile

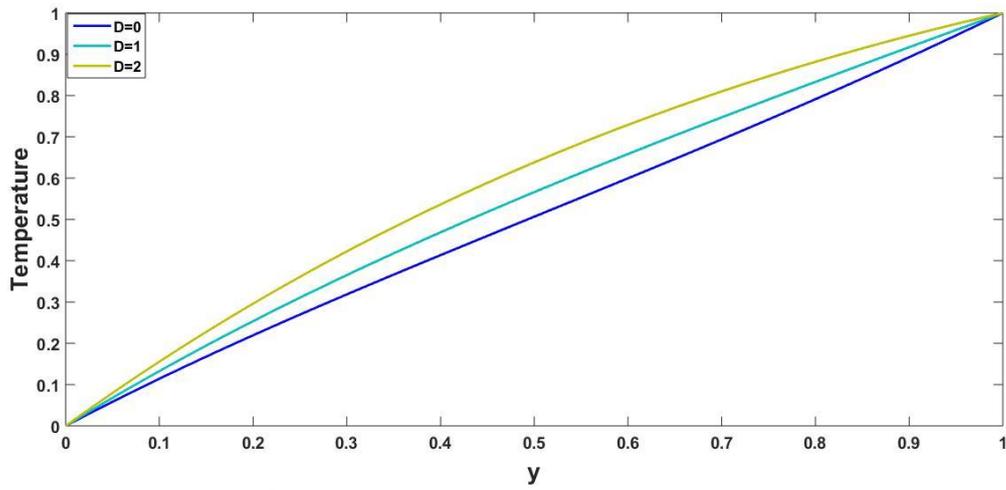


Figure 4.5: Variation of thermal radiation on Temperature profile

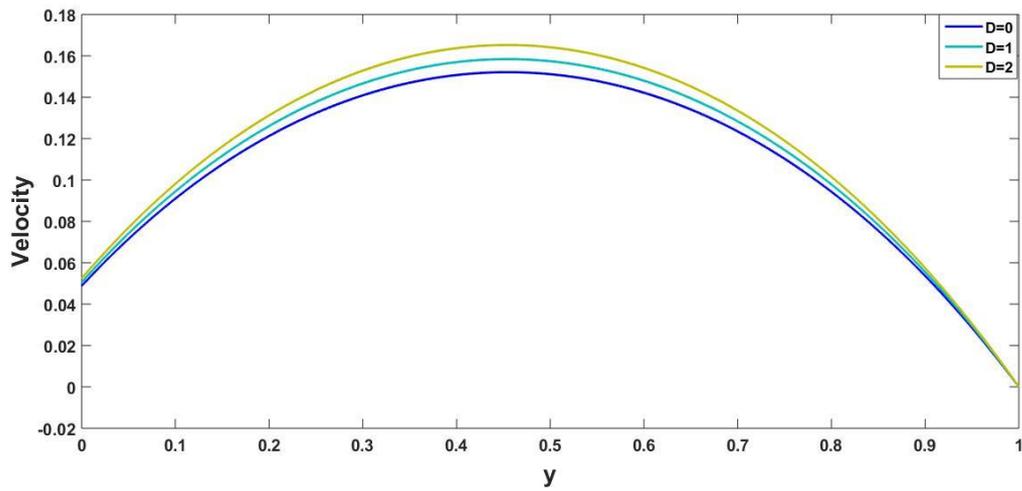


Figure 4.6: Variation of thermal radiation on Velocity profile

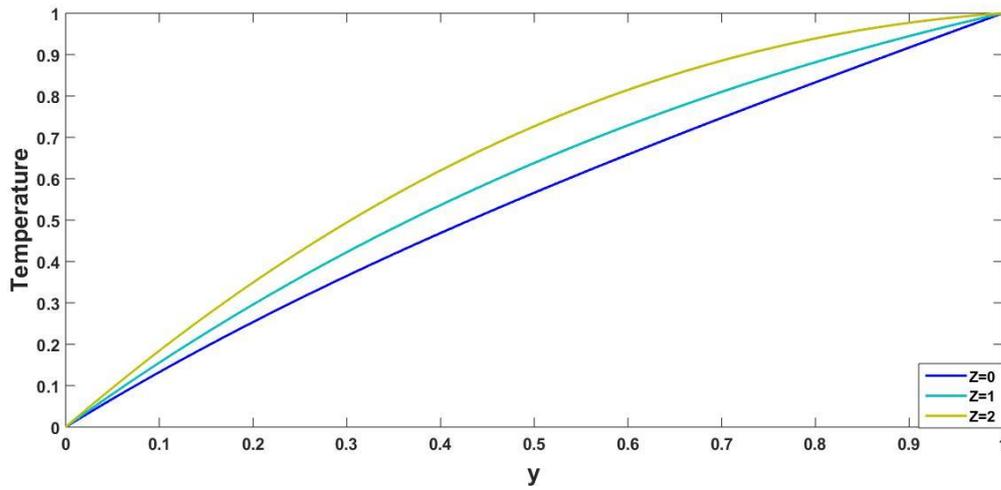


Figure 4.7: Variation of heat source/sink on temperature profile

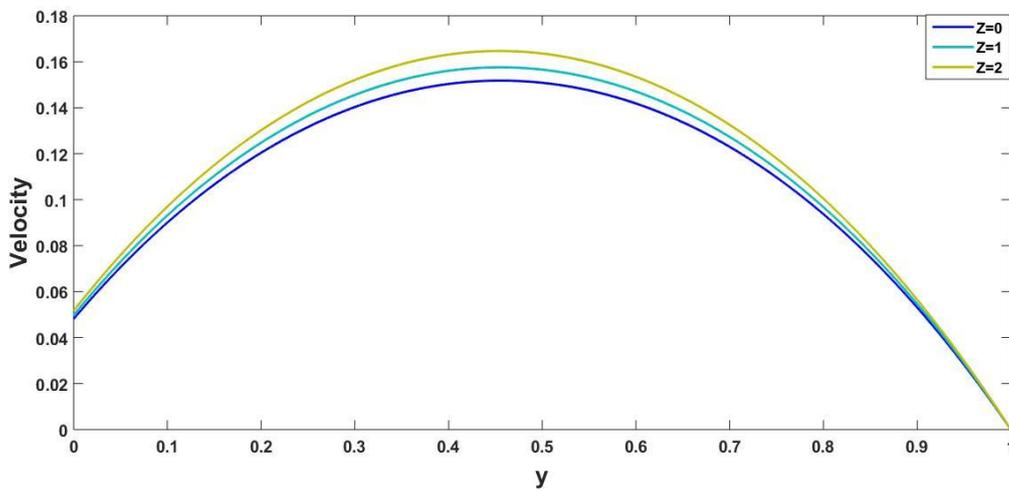


Figure 4.8: Variation of heat source/sink on Velocity profile

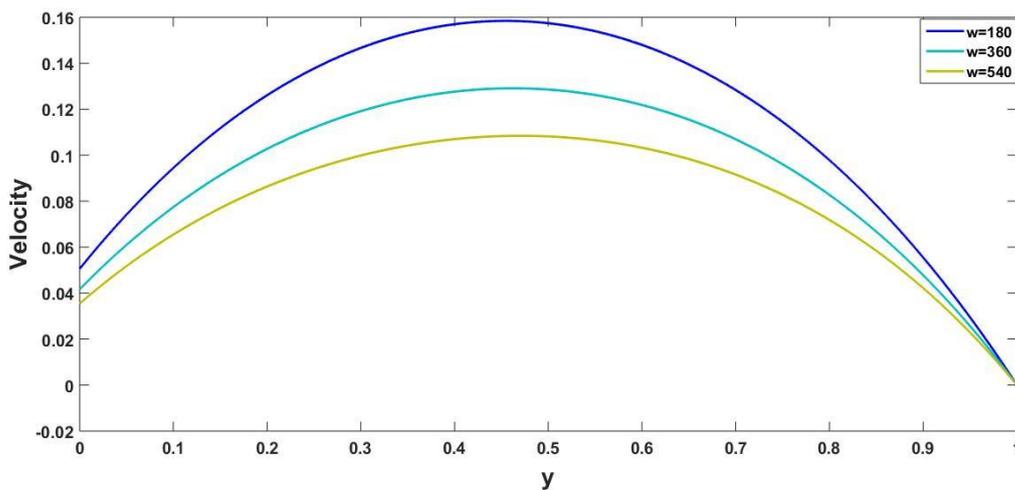


Figure 4.9: Variation of oscillation parameter on Velocity profile

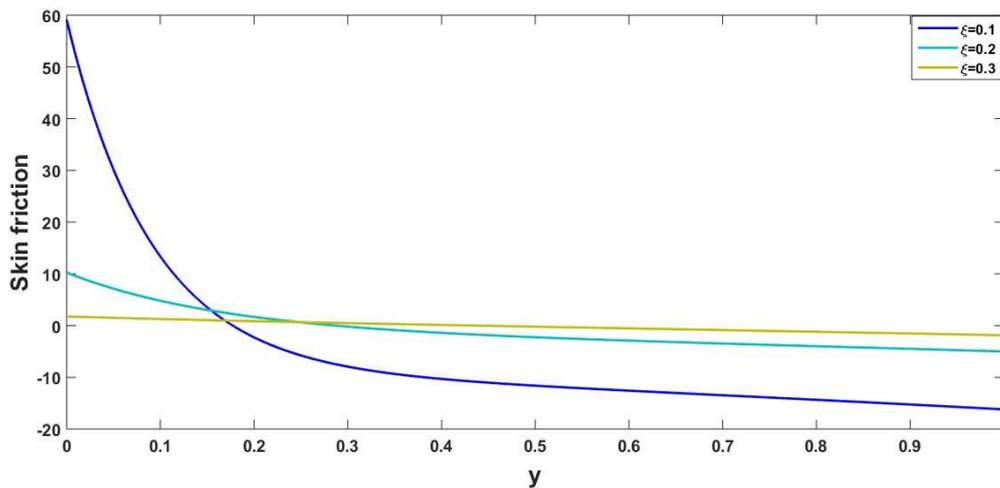


Figure 4.10: Variation of Casson parameter on Skin friction

## 5.0 CONCLUSION

The evaluation of Casson fluid and heat source/sink effects on magnetized oscillatory flow along a permeable channel saturated with porous medium is performed in the optically thin thermal radiation regime. The solutions of the non-dimensional equations have been derived and the impacts of pertinent embedded parameters dictating the flow pattern have been illustrated graphically and discussed. The Casson fluid model is an easier model that adequately explains the physiological and peristaltic flows of non-Newtonian form. The summary of the key findings from this research is highlighted below:

- i. The application of Casson fluid parameter is revealed to substantially suppress the fluid velocity
- ii. The heat source/sink parameter is envisaged to enhance the fluid temperature and velocity respectively
- iii. The fluid velocity is encouraged by higher values of porous medium parameter while a counter attribute happens for increasing thermal radiation
- iv. The effect of Casson fluid parameter is observed to raise the shear stress at the cold plate while a reverse attribute happens in the heated wall

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### Appendix

$$m_1 = \frac{sPr + \sqrt{(sPr)^2 - 4Pr(D+Z-i\omega)}}{2} \quad m_2 = \frac{-sPr - \sqrt{(sPr)^2 - 4Pr(D+Z-i\omega)}}{2}, \quad m_3 = \frac{\frac{s}{a_1} + \sqrt{\left(\frac{s}{a_1}\right)^2 - \frac{4}{a_1}\left(Ha^2 + \frac{1}{Da} + i\omega\right)}}{2}$$

$$m_4 = \frac{-\frac{s}{a_1} - \sqrt{\left(\frac{s}{a_1}\right)^2 - \frac{4}{a_1}\left(Ha^2 + \frac{1}{Da} + i\omega\right)}}{2}, J_1 = \frac{1}{e^{m_2} - e^{m_1}}, J_2 = \frac{1}{e^{m_2} - e^{m_1}}$$

$$L_3 = \frac{\lambda}{\left(Ha^2 + \frac{1}{Da} + i\omega\right)}, \quad L_4 = -\frac{GrJ_1}{m_1^2 + sm_1 - \left(Ha^2 + \frac{1}{Da} + i\omega\right)}$$

$$L_5 = -\frac{GrJ_2}{m_2^2 + sm_2 - \left(Ha^2 + \frac{1}{Da} + i\omega\right)}, \quad a_1 = 1 + \frac{1}{\xi}, \quad a_2 = L_3 + L_4 + L_5, \quad a_3 = m_1\gamma L_4 + m_2\gamma L_5,$$

$$a_4 = L_3 + L_4 e^{m_1} + L_5 e^{m_2}, \quad L_2 = -\frac{\left(a_4 + \frac{(m_4\gamma - 1)e^{m_3}}{1 - m_3\gamma}\right)}{\left(e^{m_4} + \frac{(m_4\gamma - 1)e^{m_3}}{1 - m_3\gamma}\right)},$$

$$L_1 = \frac{L_2(m_4\gamma - 1) + a_3 - a_2}{1 - m_3\gamma}$$