

Mixed convection flow of heat generating/absorbing fluid in a vertical channel with asymmetric heating condition

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ABSTRACT

The theoretical analysis is performed to determine the exact solution of mixed convection flow of a heat generating/absorbing fluid with nonlinear density variation with temperature (NDT) using nonlinear Boussinesq approximation along a vertical channel. The flow is affected by asymmetric wall heating. It is concluded from the results that, the action of heat sink parameter is to promote the fluid velocity towards the left plate then towards the right, which is as a result of the asymmetric wall heating condition. However, the fluid temperature demonstrates a deteriorating tendency in the presence of heat sink effect. Additionally, in the involvement of NDT, the amount of heat transfer substantially lowered at the both plates.

Keywords: Mixed convection, heat sink, nonlinear Boussinesq approximation, nonlinear density variation with temperature (NDT), asymmetric wall heating.

1.0 INTRODUCTION

The buoyancy force created from different temperature regions and lid-driven force from the walls movement taking place in the same system can be acknowledged as mixed convection. Mixed convection is interrelated with Richardson number, Ri , which originates from Grashof number and Reynolds number. Many studies agreed that altering Ri can modify the types of convection in investigated and related factors related to various cases such as home ventilation, fire prevention, solar collector, drying technologies, and chemical processing, as reported by (Saedodin *et al.* 2013, Mandal *et al.* 2020). According to Lavine (1988), mixed convection can be used for the dispersion of chemical contaminants in various processes in chemical industry and in transporting of heated or cooled fluids and so forth. In recent years, a number of studies have been conducted in this field. In this context, Omokhuale *et al.* (2024) recently investigated the impact of viscous dissipation on magnetized mixed convection flow with nanofluid in an oscillating system. From their computational analysis, it was revealed that, mixed convection and viscous dissipation effects are envisaged to propel the fluid velocity. Ojemeru and Onwubuya (2023) described the analysis of steady mixed convection flow of Arrhenius-controlled, incompressible and electrically conducting fluid along an isothermally heated superhydrophobic microchannel due to heat source/sink. Ahmad *et al.* (2023) evaluated the effects of a pressure gradient that was exponentially increasing or decreasing on the transient mixed convection flow of viscous reactive fluids in a vertical tube between two concentric tubes with $r = 0$ and $r = b$. The nonlinear partial differential equation governing the flow formation are solved using the implicit finite difference form where all time differentials were calculated using the forward

difference formula. Hamza *et al.* (2023) analytically and numerically examined the implication of time-dependent hydromagnetic flow of an exothermic fluid in a vertical channel regulated by convective heating and Navier slip conditions using homotopy perturbation method and implicit finite difference scheme. Hindebu *et al.* (2022) analyzed the hydrodynamic and heat transfer problems of unsteady mixed convection flow of nanofluid in a micro-channel filled with porous material. Ajibade and Umar (2021) scrutinized the effects of viscous dissipation on a steady mixed convection Couette flow of heat generating/absorbing fluid. Jafar *et al.* (2022) reported that the buoyant force has a substantial impact on the flow field for vertical or inclined surfaces. Hamza and Abdulsalam (2022) investigate the Influence of chemical kinetic exponent on transient mixed convective hydromagnetic flow in a vertical channel with convective boundary condition. Halim and Noor (2021) Scrutinized Mixed convection flow of nanofluid near stagnation point along a vertical stretching.

The use of linear Boussinesq approximation (linear density variation with temperature, LDT) approach is possible by truncating Taylor's series expansion of density with temperature after the second term. This approximation will remain accurate when the temperature difference between the channel walls is tiny. However, when the temperature difference between the channel walls is considerably huge, the mathematical simulation using a linear density temperature variation becomes incorrect. To this end, Goren (1966) established the condition for using the quadratic-density-temperature variation (nonlinear), which is, $\Delta\rho = -\rho\gamma(T - T_s)^2$. To demonstrate the advantages of this concept, Hamza *et al.* (2024a, 2024b), quite recently, analytically and numerically

investigated the impacts of magnetized free and mixed convection flows through a superhydrophobic microchannel affected by NDT. They established from their results that, the presence of nonlinear density variation with temperature (NDT) influenced by heat source drastically enhance the thermal fluid and velocity gradients in the channel. Vajravelu and Sastri (1977) scrutinized the free convection fully developed laminar flow in a vertical channel restricted to two infinite up facing parallel plates using the NDT. They found that the use of NDT leads to an increase in temperature and velocity profiles. Jha and Gwandu (2019) provided a comprehensive analysis on the exact solution for a buoyancy-induced flow of electrically conducting fluid in a vertical slit micro-channel with super-hydrophobic slip and temperature jump using NDT. The inner surface of one of the micro-channel surfaces is intentionally modified so that it demonstrates super-hydrophobic velocity slip and temperature jump. They concluded that Velocity and flow rate are higher in case of non-linear density variation with temperature in comparison to linear density variation with temperature. In a separate work, Jha and Oni (2019) derived an exact solution to depict the effect of NDT on mixed convection flow equipped with asymmetric heating conditions in a vertical channel. Other fundamental studies on the performance of NDT on various flow characteristics can be seen in Partha (2010) and Kameswaran et al. (2016). In all of the above references, none of the authors discussed the effect of heat source or sink.

Nowadays, researching the effect of heat generation or absorption on fluid flow is extremely basic in many engineering, technological and physical problems of exothermic or endothermic fluid response, and these effects are crucial in monitoring the heat transference. Heat generation/absorption effects have received significant attention owing to its fruitful applications in areas such as thermal insulation, drying engineering processes, chemical factories, coolers and heaters of electrical/mechanical devices, storage of nuclear waste and solid matrix heat exchangers amongst others (Gambo *et al.* 2021). Several scholars looked into the impact of heat generating/absorbing fluid on various flow regimes. Several scholars have investigated the effects of heat-generating/absorbing fluid on various flow regimes. In this context, Ojmeri *et al.* (2024) recently published an analytical investigation of a chemically reactive hydromagnetic fluid affected by heat generation/absorption flowing across a superhydrophobic microchannel that is heated alternatively. In addition, Ojmeri and Onwubuya (2023) described the analysis of steady mixed convection flow of Arrhenius-controlled chemical reaction and an exothermic fluid along an isothermally heated superhydrophobic microchannel due to heat source/sink. In another paper, Ojmeri and Hamza (2022) presented a theoretical investigation of a

chemically reactive fluid in a MHD natural convection flow blended with heat source and sink effects employing homotopy perturbation approach. They concluded that raising the viscous heating parameter significantly encourage the fluid motion and the volume flow rate respectively. Further, Oni and Jha (2019) investigated the effects of a heat source and sink fluid on natural convection confined to an upstanding annulus with time-periodic heating boundary conditions. They discovered that heat generation and absorption parameters affect velocity distribution, periodic temperature profile, Nusselt number, and frictional force at the cylinder walls, respectively. Gambo and Gambo (2020) extensively scrutinized the effects of fully developed natural convective flow of heat generating/absorbing fluid in an open-ended vertical concentric annulus with a magnetic field effect. Their findings show that by carefully selecting appropriate values for the heat generation or absorption parameter and the Hartmann number, they may be used to control the temperature and velocity profiles. Chu *et al.* (2020) investigated the nonlinear thermal radiation and heat sink/source of a bidirectional periodically moving surface driven flow in a rate type nanofluid containing a gyrotactic microbe. Zhao *et al.* (2021) highlighted the effects of heat generation/absorption, dissipation, radiative heat flux and joule heating on the mixed convective entropy optimized nanomaterial MHD flow of Ree-Eyring fluid formed by two rotating disks. The consequences of thermal radiation, a heat source, and an induced magnetic field on the natural convective flow of a couple stress fluid in a flux-isothermal vertical channel have been evaluated by Hasan *et al.* (2020) using the method of an indeterminate coefficient. Parthiban and Pasad (2023) outlined a theoretical investigation of radiative-convection effects on MHD fluid flow in a heated square enclosure having a non-Darcy square cavity in the coexistence of the Hall effect and the heat source or sink.

From the preceding discourse, not much work has been done on the effect of mixed convection with NDT involving flow reversal regulated by heat sink impact. This research attempts to present a unique combination by building on the work of Jha and Oni (2019) by considering the effects of heat absorbing fluid on mixed convection flow with NDT affected by asymmetric wall heating condition. It is hoped that the outcome of the study will be useful in industrial applications, engineering and biomedical sciences such as for improving heat transfer efficiency, chemical synthesis, heating and energy generation, designing efficient energy conversion processes, to mention a few. Additionally, this present paper would serve as a basis for checkmating the accuracy of other numerical techniques in studies involving flow reversal.

2.0 FORMULATION OF THE PROBLEM

Envisage a steady fully developed mixed convection flow of an incompressible viscous fluid in a vertical channel formed by two infinite vertical parallel plates affected by constantly applied pressure gradient and buoyancy force with nonlinear Boussinesq approximation. The channel wall $y=H$ is assumed to be heated to temperature T_2 greater than the surrounding fluids having temperature T_0 and the wall ($y=0$) is heated to a temperature T_1 such that $T_2 > T_1$. Schematic diagram of the present problem is described in Figure 1, where the x -axis is the along the flow direction parallel to the gravitational acceleration g but in the opposite direction. The mathematical model used to capture the nonlinear Boussinesq approximation term is derived by expanding Taylor's series up to second-degree polynomial as seen in (Sinha 1969, Kameswaran *et al.* 2016). Following the work Jha and Oni (2019) where they used nonlinear Boussinesq approximation, and assuming that the fluid is affected by heat source/sink effect, the resultant governing

dimensional equations describing the current flow problem can be expressed as:

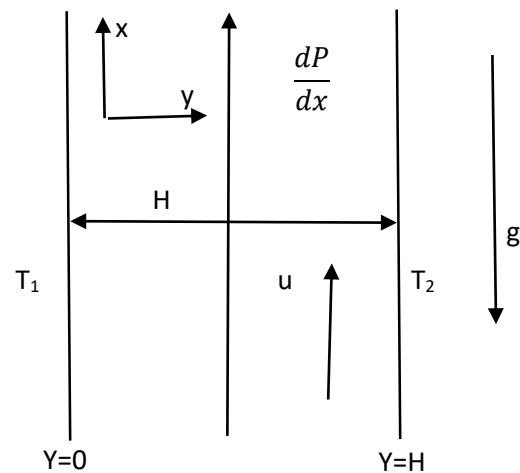


Figure 1: Physical configuration of the Flow

$$\nu \frac{d^2 u}{dy^2} + g \rho_0 \left[\beta_0 (T - T_0) + \beta_1 (T - T_0)^2 \right] = \frac{dp}{dx} \quad (1)$$

$$\frac{d^2 T}{dy^2} - \frac{Q(T - T_0)}{\rho C_p} = 0 \quad (2)$$

Subject to the following boundary conditions

$$\left. \begin{aligned} u = 0, T = T_1, \text{ at } y = 0 \\ u = 0, T = T_2, \text{ at } y = H \end{aligned} \right\} \quad (3)$$

The relevant non-dimensional quantities and parameters for all the problems are:

$$\left. \begin{aligned} U = \frac{u}{u'}, Y = \frac{y}{H}, \alpha = \frac{dp}{dx}, \theta = \frac{T - T_0}{T_2 - T_0}, \text{Re} = \frac{u' H}{\nu} \\ P = \frac{p}{\rho u'^2}, \sigma_T = \frac{T_1 - T_0}{T_2 - T_0}, Gr = \frac{g \beta_0 (T_2 - T_0)}{\nu^2}, \\ X = \frac{x}{H \text{Re}}, C = \frac{\beta_1 (T_2 - T_0)}{\beta_0}, S = \frac{QH^2}{k} \end{aligned} \right\} \quad (4)$$

Using (4), equations (1) to (3) becomes

$$\frac{d^2 U}{dY^2} + \frac{Gr}{\text{Re}} \left[\theta + C \theta^2 \right] = \alpha \quad (5)$$

$$\frac{d^2 U}{dY^2} - S \theta = 0 \quad (6)$$

Restricted to the following boundary conditions

$$\left\{ \begin{aligned} U = 0, \theta = \sigma_T, \text{ at } Y = 0 \\ U = 0, \theta = 1, \text{ at } Y = 1 \end{aligned} \right. \quad (7)$$

3.0 SOLUTION PROCEDURE

The exact solutions for the temperature and velocity distributions have been determined as follows:

$$\theta = A \exp(Y\sqrt{S}) + B \exp(-Y\sqrt{S}) \quad (8)$$

$$U = \frac{\alpha}{2} [Y^2 - Y] + \frac{Gr}{Re} [D_1 \exp(Y\sqrt{S}) + D_2 \exp(-Y\sqrt{S}) + D_3 Y + D_4] + \dots \quad (9)$$

$$+ \frac{Gr}{Re} [D_5 \exp(2Y\sqrt{S}) + D_6 \exp(-2Y\sqrt{S}) + D_7 Y^2 + D_8 Y + D_9]$$

The rate of heat transfer and shear stress have also computed as:

$$\left. \frac{d\theta}{dY} \right|_{Y=0} = A\sqrt{S} - B\sqrt{S} \quad (10)$$

$$\left. \frac{d\theta}{dY} \right|_{Y=1} = A\sqrt{S} \exp(\sqrt{S}) - B\sqrt{S} \exp(-\sqrt{S}) \quad (11)$$

$$\left. \frac{dU}{dY} \right|_{Y=0} = -\frac{\alpha}{2} + \frac{Gr}{Re} [D_1\sqrt{S} - D_2\sqrt{S} + D_3] + C \frac{Gr}{Re} [2D_5\sqrt{S} - 2D_6\sqrt{S} + D_8] \quad (12)$$

$$\left. \frac{dU}{dY} \right|_{Y=1} = \frac{\alpha}{2} + \frac{Gr}{Re} [D_1\sqrt{S} \exp(\sqrt{S}) - D_2\sqrt{S} \exp(-\sqrt{S}) + D_3] + \quad (13)$$

$$C \frac{Gr}{Re} [2D_5\sqrt{S} \exp(2\sqrt{S}) - 2D_6\sqrt{S} \exp(-2\sqrt{S}) + 2D_7 + D_8]$$

4.0 RESULTS AND DISCUSSION

The performance assessment of heat source/sink impacts on mixed convection flowing through an immeasurable upstanding plate has been performed. The flow is instigated by buoyancy-induced constant pressure gradient along an upward facing plate. In order to point out the effects of physical parameters such as; heat sink parameter S , nonlinear density variation with temperature (NDT) C , asymmetric heating parameter σ_t , and mixed convection parameter Gr , on the flow behaviours, computation of the flow fields is carried out. The influences of the major controlling parameters on the temperature and velocity distributions have been presented and discussed in Figures 3 to 7. The main default values selected for this analysis as they relate to real life applications are $S = 0.2$, $\alpha = 0.5$, $C = 0.02$ and $Gr=200$. The graphical comparison between the work of Jha and Oni (2019) and the present investigation establishes an excellent agreement as shown in figure 2, when the heat sink parameter is absent. Figure 3a and b displays the flow patterns in the temperature and velocity distributions respectively when heat sink parameter is varied. It is seen from figure 3a that, for mounting level of heat sink parameter, the fluid velocity is expanded towards the left wall before it decays towards the right wall. However, a point of intersection is recorded at the centre of the vertical channel where the parameter S is independent of flow in that region. This is attributable to the asymmetric heating condition affecting the flow

formation. On the other hand, a downwards trend is witnessed in the temperature of the fluid for higher values of heat sink parameter. This is because after the heat is taken in, the fluid becomes heavier, and the convection flow decreases, contributing to a drop in the temperature of the fluid. The deviations of Gr and C on the velocity distribution is demonstrated in figure 4. As shown in figure 4a, the function of positive Gr is to enhance the fluid flow towards the right plate while the negative Gr does the opposite. It is however observed that a point of intersection is noticed at the centre of the channel, which is due to the action of the asymmetric wall heating. The presence of NDT was deduced to improve the velocity fluid as plotted in figure 4b. The behaviour of skin friction against asymmetric wall heating parameter is demonstrated in figure 5. It is apparent that, in the involvement of NDT, the presence of growing heat sink parameter magnifies the skin friction at the lower plate while a counter attribute happens at the upper plate with a bit of intersection at the middle. With the aid of figure 6, the action of heat sink parameter versus asymmetric wall heating parameter on the rate of heat transfer is depicted. A decreasing tendency in the heat transfer rate at both plates is experienced for rising values of heat sink parameter. The implication of varying NDT against γ on the shear stress is illustrated in figure 7. It is evident that, there is a decline in the frictional force at the lower plate when the NDT is increased, whereas a contrast behaviour occurs at the upper plate.

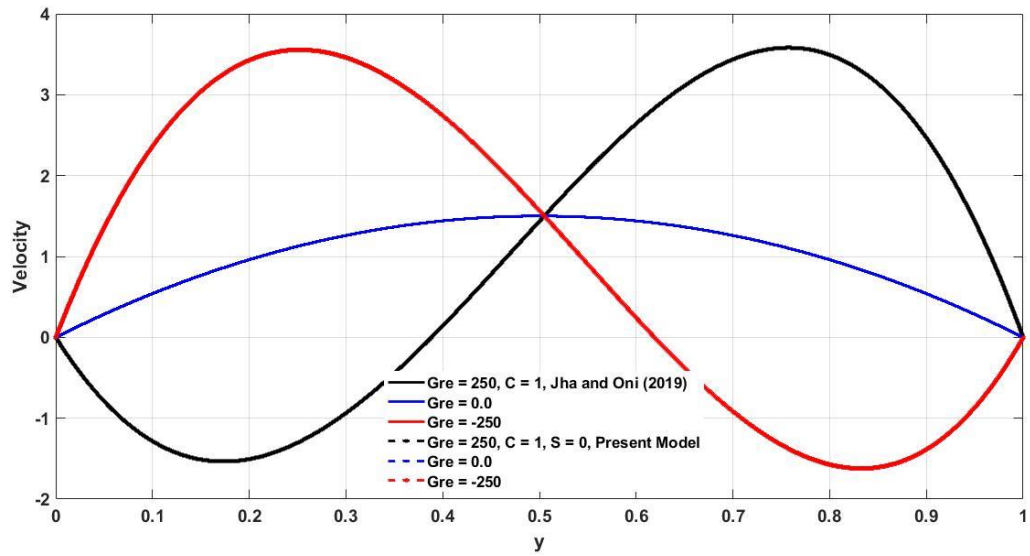


Figure 2: Comparison between the work of Jha and Oni (2019) and the present model when $S = 0$

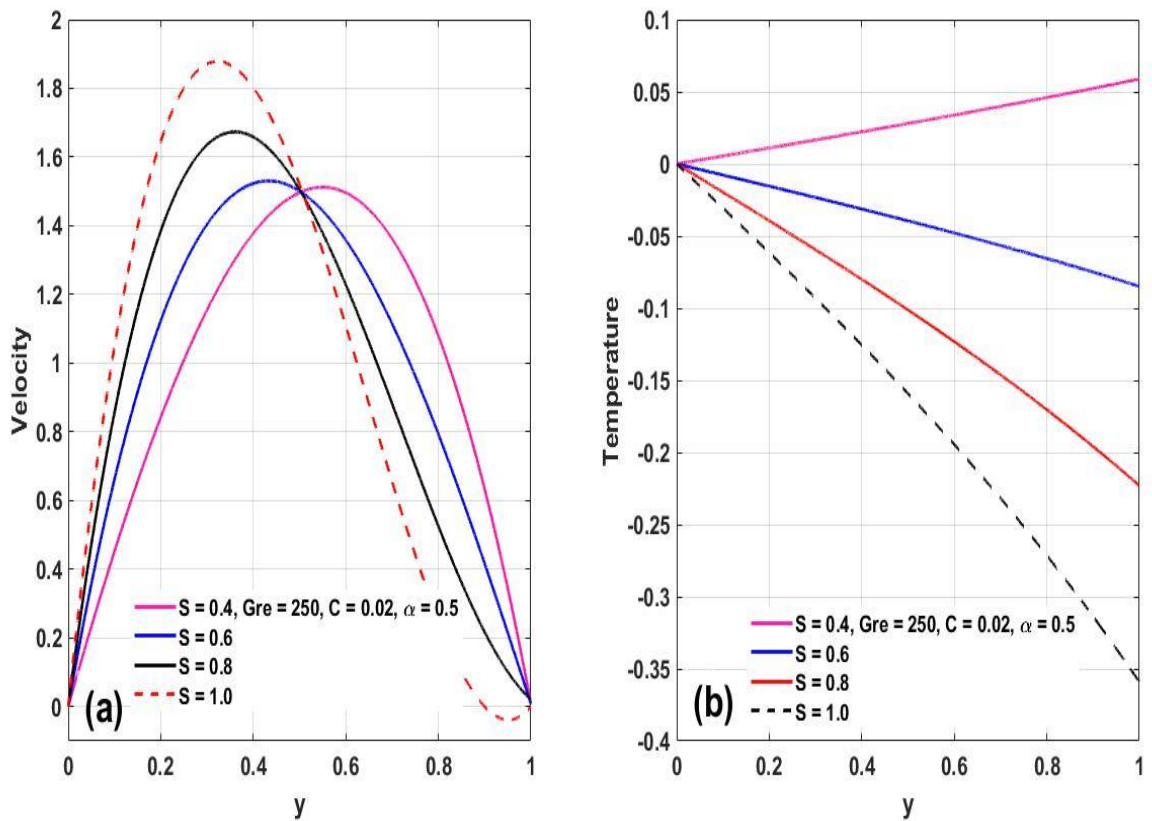


Figure 3: Velocity and Temperature profile for distinct values of S

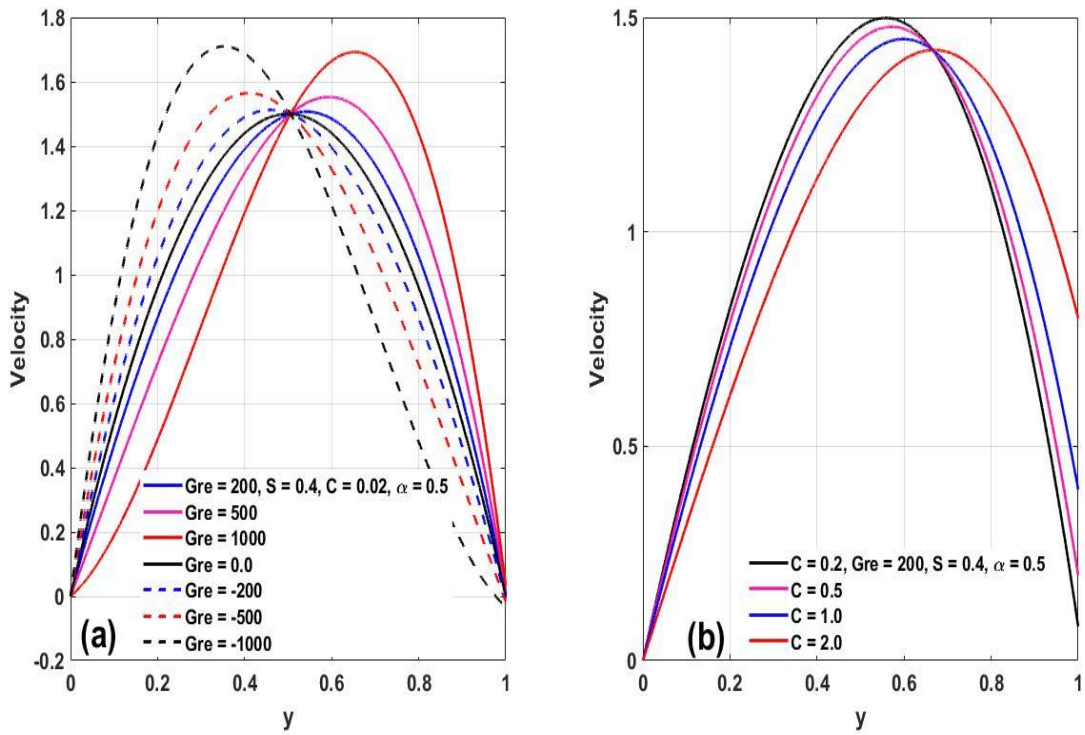


Figure 4: Influence of Gre and C on velocity distribution

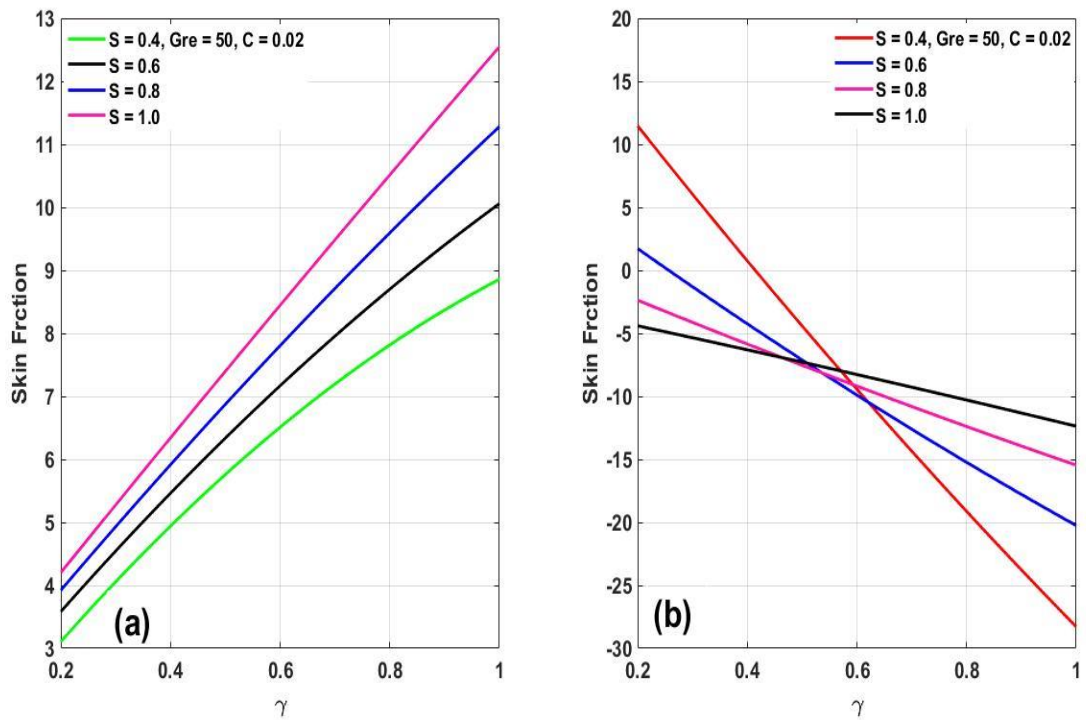
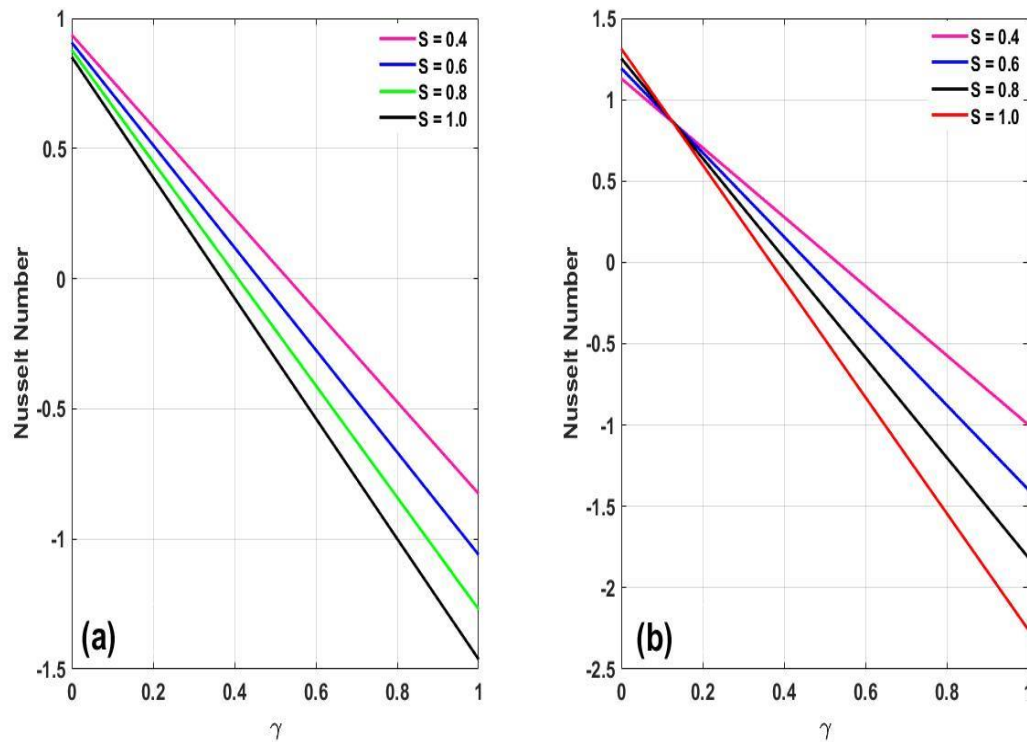
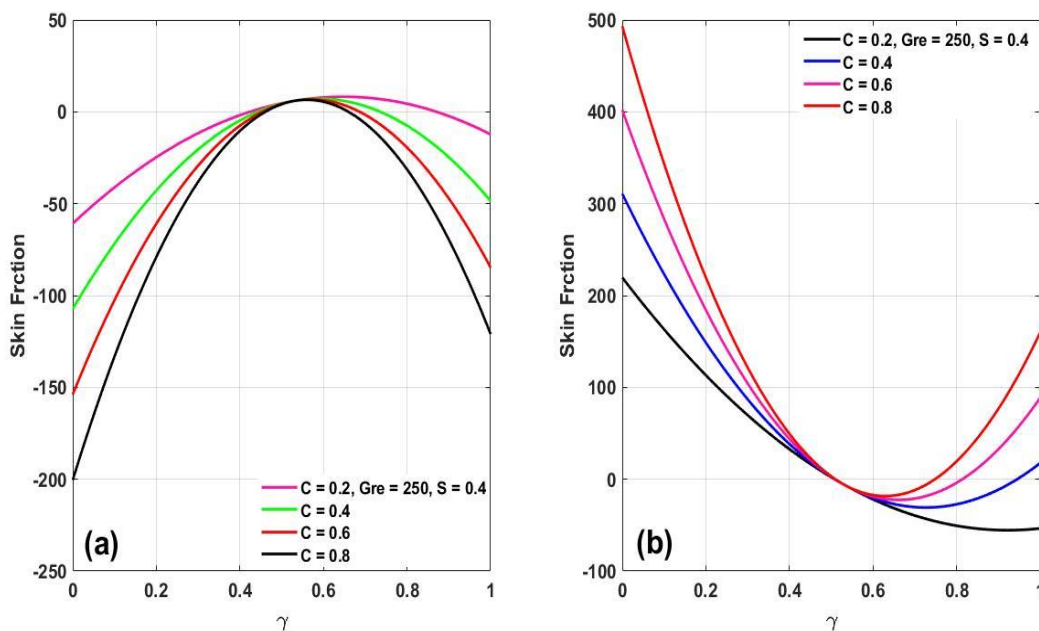


Figure 5: Skin friction against γ for different values of S

Figure 6: Nusselt number against γ for different values of S Figure 7: Skin friction against γ for distinct values of C

5.0 CONCLUSION

The steady state examination of heat generating/absorbing fluid using nonlinear Boussinesq approximation is performed on mixed convection flow in a vertical channel restricted to asymmetric wall heating condition. The governing equation containing the nonlinearity is deduced and incorporated into the momentum equation. Exact solutions are obtained

using the theory of simultaneous differential equation for fluid temperature, velocity, skin friction and rate of heat transfer. Various illustrative graphs have also been plotted to explain the impacts of major pertinent parameters embedded in the flow formation. The summary of the noteworthy findings is outlined below:

- i. It concluded that increasing heat sink parameter is seen to weaken the temperature and velocity gradients respectively.
- ii. It was observed that the function of nonlinear Boussinesq approximation is to increase the fluid velocity as well as skin-friction.
- iii. The presence of growing heat sink parameter magnifies the skin friction at the lower plate while a counter attribute happens at the upper plate
- iv. A deteriorating tendency in the heat transfer rate at both plates is experienced for rising values of heat sink parameter.
- v. The computational results obtained in this present paper consistent with the work of Jha and Oni (2019) when the heat sink parameter ignored.
- vi. In the future, the effect of variable thermal conductivity will be studied on this model.

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