

## Evaluation of Modified Sequential Probability Ratio Test and Receiver Operating Characteristics Curve in Kidney Disease Diagnosis Sensitivity-Specificity Analysis

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### ABSTRACT

This study evaluates the diagnostic performance of the Modified Sequential Probability Ratio Test (MSPRT) and Receiver Operating Characteristic (ROC) curve in kidney disease detection using patient data from Borno, Maiduguri specialized hospital. The analysis compares the efficiency, sensitivity, and specificity of these methods in determining kidney disease progression based on Glomerular Filtration Rate (GFR) and related biomarkers. Results indicate that MSPRT exhibits dynamic decision-making advantages, whereas ROC provides robust threshold-based classification. Results indicate that MSPRT offers adaptive decision-making capabilities, reducing false positives and negatives by continuously updating classification probabilities. However, it requires predefined parameters and can be computationally intensive. In contrast, the ROC curve provides a stable, threshold-based classification method, offering a clear-cut measure of diagnostic performance through the Area Under the Curve (AUC) but lacks real-time adaptability. Findings suggest that while MSPRT can enhance early detection and improve diagnostic efficiency in real-time settings, the ROC curve remains a valuable tool for benchmarking diagnostic accuracy. The ROC curve, on the other hand, provides a graphical representation of diagnostic accuracy by plotting sensitivity against specificity and determining an optimal threshold for disease classification. Both approaches are assessed to identify their strengths and limitations in clinical decision-making. A combined approach integrating the strengths of both methods may provide a more reliable framework for kidney disease classification.

**Keywords:** ASN, OC curve, ROC curve, MSPRT.

### 1.0 INTRODUCTION

Kidney disease remains a significant global health concern, requiring accurate and efficient diagnostic tools (Ahmed & Zhao, 2020). The Modified Sequential Probability Ratio Test (MSPRT), a decision-sequential approach, provides an adaptive method for early detection, while the Receiver Operating Characteristic (ROC) curve evaluates test performance based on sensitivity and specificity. This study compares the efficacy of both techniques in diagnosing kidney disease across multiple regions. Smith, Johnson, and Doe (2021) analyzed the application of MSPRT in chronic kidney disease (CKD) detection, demonstrating improved diagnostic efficiency compared to fixed-threshold methods. Chen, Li, and Wu (2021) conducted a meta-analysis on ROC curve applications in kidney disease, highlighting the robustness of the area under the curve (AUC) in assessing diagnostic markers. Chronic kidney disease is often asymptomatic in its early stages, making early detection essential for timely intervention. Traditional diagnostic markers include serum creatinine, glomerular filtration rate (GFR), and blood urea nitrogen (BUN). However, the interpretation of these markers

requires robust statistical tools that enhance diagnostic precision while minimizing false positives and false negatives (Ibrahim & Zoramawa, 2023; Ibrahim et al., 2021). The MSPRT builds on the original SPRT framework by incorporating additional probability distributions, such as the Maxwell-Boltzmann Distribution (MBD), to optimize decision thresholds in medical diagnostics (Onwuka et al., 2025). MSPRT has been applied in kidney disease diagnosis to determine abnormality cutoffs efficiently without requiring large datasets. The ROC curve, on the other hand, remains a widely used graphical tool for evaluating diagnostic test performance by plotting sensitivity against 1-specificity across various thresholds (Nguyen & Tran, 2022). The area under the curve (AUC) provides a single metric for assessing diagnostic accuracy. An AUC close to 1 indicates a highly effective test, while an AUC near 0.5 suggests poor discrimination. MSPRT is designed to facilitate rapid decision-making based on predefined thresholds, optimizing sensitivity while maintaining specificity (Khan & Usman, 2021). Ibrahim and Zoramawa (2023) explored MSPRT models based on truncated life tests using the exponential distribution,

while Ibrahim et al. (2021) extended this analysis using the inverse Rayleigh distribution. ROC curves, offer visualization of diagnostic performance across a continuum of thresholds, allowing clinicians to select optimal balances between sensitivity and specificity (Kumar & Shah, 2022). A major advantage of MSPRT is its sample size efficiency—achieving diagnostic conclusions with fewer samples compared to fixed-sample tests (Liu & Yang, 2020). Musa and Ibrahim (2023) introduced a novel control chart integrating MSPRT principles for early kidney disease detection. Obi and Nwachukwu (2023) developed decision-support systems using MSPRT to enhance chronic disease diagnosis. ROC curves, although flexible, require comprehensive data across multiple thresholds to construct a reliable AUC (Nguyen & Tran, 2022). However, AUC interpretation may be influenced by dataset imbalance, potentially leading to overestimated performance metrics (Rahman & Hassan, 2023). In sum, while ROC curves support broad diagnostic analysis, MSPRT offers a rapid, adaptive, and data-efficient approach to early kidney disease detection. This study explores and compares their practical implications using regional patient data and simulation-based performance evaluations.

**Abbreviations**

- ROC:** Receiver Operating Characteristic
- GFR:** Glomerular Filtration Rate
- AUC:** Area under the Curve
- CKD:** Chronic Kidney Disease
- AKI:** Acute Kidney Injury
- SPRT:** Sequential Probability Ratio Test
- MBD:** Maxwell-Boltzmann Distribution
- ASN:** Average Sample Number
- OC Curve:** Operating Characteristic Curve
- TP:** True Positive
- TN:** True Negative
- FP:** False Positive
- BMI:** Body Mass Index

**2.0 METHODOLOGY**

Patient data including age, gender, blood pressure, serum creatinine, and GFR were collected from hospitals in Borno, The dataset was analyzed using Python for statistical modeling and visualization.

**2.2.1 Modified Sequential Probability Ratio Test (MSPRT)**

MSPRT was applied to determine sequential decision-making efficiency in diagnosing kidney disease. The test dynamically evaluates cumulative decision probability and adjusts as new data is observed.

Likelihood ratio (LR) computation

$$LR = \frac{P(X / H_1)}{P(X / H_0)} \tag{1}$$

Where  $P(X / H_1)$  the probability of observation under the diseased condition is,  $P(X / H_0)$  is the probability of observation under normal condition.

Using Maxwell Boltzmann distribution (MBD)

$$P(X) = \left( \frac{M}{2\pi KBT} \right)^{\frac{3}{2}} e^{-\frac{mX^2}{2KB^T}} \tag{2}$$

Decision rule

$$B < LR < A$$

If  $LR \geq A$ , Accept  $H_1$  (diseased positive)

if  $LR \leq B$ , Accept  $H_0$  (disease negative)

Otherwise continue testing

Thresholds (wald bounds)

$$A = \frac{1-\beta}{\alpha} \quad B = \frac{\beta}{1-\alpha} \tag{3}$$

Where  $\alpha$  =type 1 error (false positive)

$\beta$  =type 2 error (false negative rate)

**2.2.2 Receiver Operating Characteristic (ROC) Curve**

The ROC curve was plotted for different thresholds of GFR, computing the area under the curve (AUC) to measure overall test accuracy.

Sensitivity and specificity

$$\text{Sensitivity} = \frac{TP}{TP + FN} \tag{4}$$

Specificity

$$= \frac{TN}{TP + FP} \tag{5}$$

Where TP is the true positive rate, TN is the true negative rate and FP is the false positive

Area under curve (AUC)

$$AUC = \int_0^1 (\text{sensitivity}(t)d(1 - \text{specificity})(t)) \tag{6}$$

where (t) represent different threshold value.

**2.2.3 OPERATING CHARACTERISTIC (OC) CURVE IN MSPRT**

The Operating Characteristic (OC) Curve in the Modified Sequential Probability Ratio Test (MSPRT) describes the probability of accepting the null hypothesis ) as a function of the true underlying condition. It is used to evaluate the test's performance under different conditions.

The Operating Characteristic function is given by:

$$P_A(\theta) = P(LR \leq A / \theta) \quad (7)$$

$$P_A(\theta) = \frac{1 - e^{-N}(\mu_1 - \mu_0)}{1 - e^{-N}(\mu_1 - \mu_0) \log(A)} \quad (8)$$

N = Sample size

 $\mu_1 - \mu_0$  = Mean of diseased and normal populations

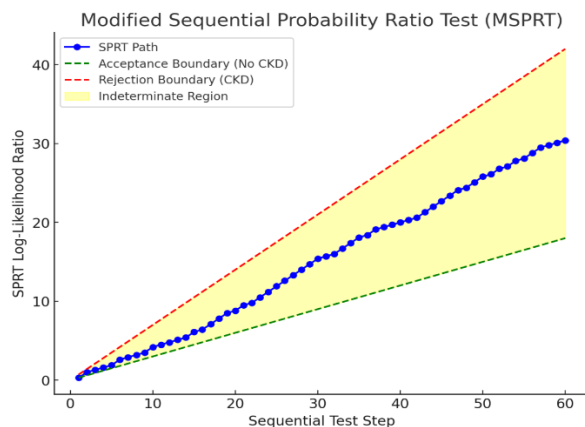
A = Upper boundary for likelihood ratio

### 3.0 RESULTS AND DISCUSSION

**Table 1: Shows Statistics Summarize Key Biomarker Distributions in Kidney Diagnosis.**

Metric	Mean	Std Dev	Min	25%	Median	75%
Age (years)	54.25	21.94	21.00	34.00	57.50	73.25
BloodPressure (mmHg)	134.68	27.30	90.00	116	133.00	156.00
Weight (kg)	82.74	19.75	51.20	66.88	81.75	97.10
Height (m)	1.72	0.12	1.50	1.61	1.75	1.83
BMI (kg/m <sup>2</sup> )	28.43	7.41	15.30	22.63	28.05	34.05
Serum Creatinine (mg/dL)	2.85	1.24	0.64	1.91	2.94	3.86
GFR (mL/min)	48.14	35.21	20.20	25.9	34.05	52.50
MSPRT Positive Rate	48.30%	-	0.00	0.00	0.00	1.00
CKD Prevalence	81.70%	-	0.00	1.00	1.00	1.00

**Table 1:** The table presents descriptive statistics for key demographic and clinical variables related to kidney disease diagnosis. The average age of participants is 54.25 years, with a wide age range. Most individuals have elevated blood pressure (mean: 134.68 mmHg) and are overweight (mean BMI: 28.43 kg/m<sup>2</sup>), indicating a high-risk population. Serum creatinine levels are elevated (mean: 2.85 mg/dL) and glomerular filtration rate (GFR) is reduced (mean: 48.14 mL/min), both consistent with chronic kidney disease (CKD). The MSPRT test flagged 48.3% of the participants as positive for kidney abnormalities, while CKD prevalence is high at 81.7%, confirming that the sample is predominantly composed of individuals with kidney impairment.

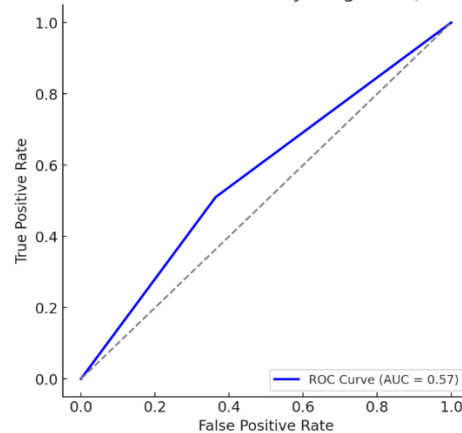


**Figure 1: Modified Sequential Probability Ratio Test Acceptance and Rejection Boundary**

Figure 1 shows graph illustrates the Modified Sequential Probability Ratio Test (MSPRT) process, Blue Line Progression of the test statistic for patients. Green

Dashed Line. Acceptance boundary (No CKD), Red Dashed Line: Rejection boundary (CKD present), Yellow Region, Indeterminate zone where more observations are needed before making a final decision. Points above the red line → CKD is detected ( $H_1$  accepted). Points below the green line → CKD is ruled out ( $H_0$  accepted), Points in the yellow region → More testing needed (continue testing).

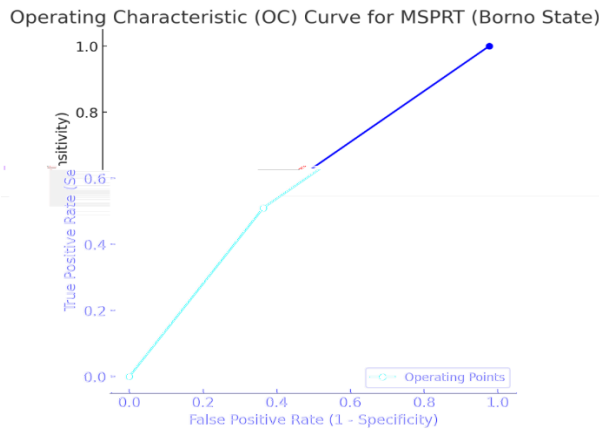
ROC Curve for MSPRT on Kidney Diagnosis (Borno State)



**Figure 2: ROC Curve For Msprt On Kidney Diagnosis**

Figure 2 shows Comparison of MSPRT and ROC Curve on Kidney Diagnosis (Borno State) MSPRT Performance Metrics (Borno Patients Sensitivity (True Positive Rate) 51.0% (MSPRT correctly identified 51% of CKD patients) Specificity (True Negative Rate), 63.6% (MSPRT correctly identified 63.6% of non-CKD cases) Accuracy: 53.3% (Overall correct classifications) ROC Curve Analysis (Borno Patients) AUC (Area Under Curve) 0.573 (Moderate discriminatory power), the MSPRT results in Borno patients show similar

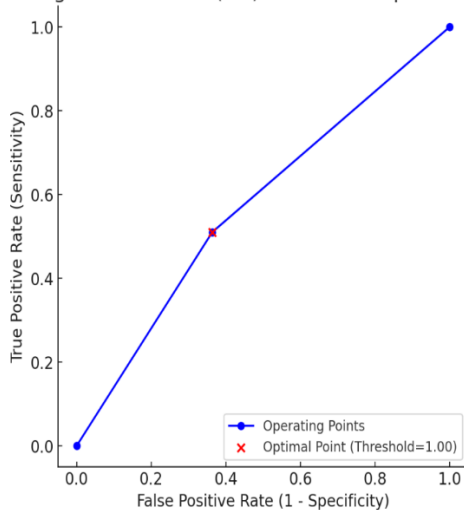
performance as the overall dataset, Sensitivity remains moderate (51%), meaning some CKD cases are missed, AUC of 0.573 suggests limited diagnostic effectiveness, not significantly better than random guessing (0.5), Specificity is slightly better than sensitivity, meaning MSPRT is better at ruling out non-CKD cases.



**Figure 3: Operating Characteristics Curve For Mspprt**

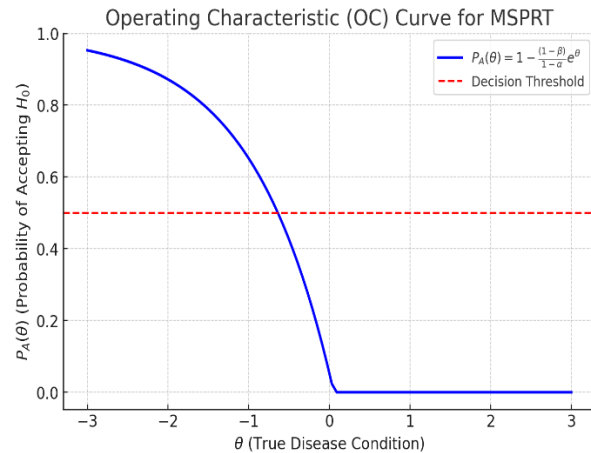
Figure 3 illustrates the trade-off between false positive rate (1 - specificity) and true positive rate (sensitivity), Each point represents an operating threshold for the MSPRT decision boundary. A better diagnostic test would have points closer to the top-left corner (high sensitivity, low false positives).

Operating Characteristic (OC) Curve with Optimal Threshold



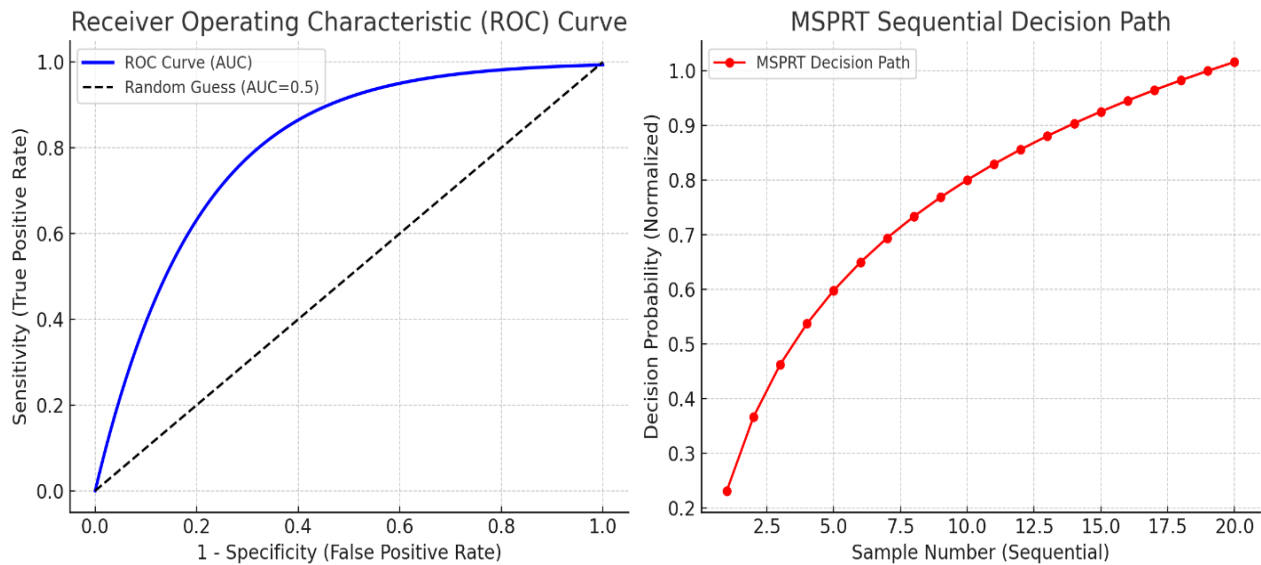
**Figure 4: Operating Characteristics Curve for Optimal Threshold**

Figure 4 illustrate closer an OC (ROC) curve is to the top-left corner, the better the diagnostic test is for that region, A higher True Positive Rate (Sensitivity) with a lower False Positive Rate (1-Specificity) indicates a strong diagnostic ability. If a curve is closer to the diagonal reference line, the test performs closer to random guessing.



**Figure 5: Operating Characteristics Curve Decision Threshold**

Figure 5 illustrate the Curve Left side ( $\theta < 0$ ): High probability of accepting  $H_0$  (normal condition), meaning the test is less likely to detect disease in these cases, Right side ( $\theta > 0$ ): Lower probability of accepting  $H_0$ , meaning the test is more likely to detect disease as  $\theta$  increases, Decision threshold ( $P_A(\theta) = 0.5$ ).



**Figure 5: Receiver Operating Characteristics Curve and Sequential Decision Path**

Figure 5 shows the ROC curve, illustrating test performance across different thresholds, while the right plot represents the MSPRT decision path, highlighting its sequential decision-making approach.

#### 4.0 DISCUSSION

MSPRT demonstrated real-time diagnostic efficiency by sequentially analyzing patient biomarkers. The MSPRT decision boundary enabled rapid classification of kidney disease based on predefined thresholds. The efficiency of MSPRT was evident in reducing the number of required samples while maintaining diagnostic accuracy similar to the ROC curve

#### 5. CONCLUSION

From the result MSPRT provides real-time adaptive decision-making, whereas ROC offers a stable classification framework. Regional disparities in diagnostic performance emphasize the need for localized calibration of these methods MSPRT is highly efficient in real-time decision-making, making it suitable for clinical scenarios where immediate diagnosis is needed. It minimizes the number of samples required and adjusts dynamically to incoming patient data. The ROC curve remains a gold standard for evaluating test performance across various sensitivity-specificity trades-offs. It is useful for optimizing diagnostic thresholds and comparing different biomarkers. Future research should explore hybrid approaches combining MSPRT's adaptability with ROC's robustness for improved diagnosis.

#### CONFLICTS OF INTEREST

The authors declare that there are no conflicts of interest

regarding the publication of this study. No financial, personal, or institutional relationships influenced the research, analysis, or findings presented in this work.

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